Online Appendix: Global Hegemony and Exorbitant Privilege *

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B Calibration and Numerical Results

B.1 Additional assumptions for quantitative exercise

We set $\tau = 1$ and interpret χ as debt capacity in dollars. It is useful to allow for a factor for the effectiveness of military spending, α. The parameter α can be thought as pinning down the units of our model, e.g. otherwise it is not clear whether m_{it} is measured in dollars, billions of dollars, hundreds of billions, etc. We also allow for a discount factor β, so we can talk about the natural real rate $\bar{r}_f = -\log \beta$. From now on, we consider the case with $A = 0$ throughout, i.e. any military advantage is endogenous.

These additional assumptions mean that the contest function becomes

$$w(m_i, m_{-i}) \quad = \quad F(\alpha(m_i - m_{-i})), \tag{B.1}$$

and the international investors maximize

$$E\left[c_{i0} + \kappa c_{i1}\right], \ \kappa = \{\beta, \theta \times \beta\} \tag{B.2}$$

B.2 Multiplicity and fragility thresholds with additional assumptions

Then, the proof of Proposition 2 needs to be modified as follows. The bond price equals

$$q_{it}(m_{it}, m_{-it}) \quad = \quad \beta\left(1 - \phi + \phi\theta w(m_{it}, m_{-it})\right), \tag{B.3}$$

$$\quad = \quad \beta\left(1 - \phi\theta F(\alpha(m_{it} - m_{-it})\right). \tag{B.4}$$

A steady-state is a zero of the function

$$h(z) \quad = \quad \delta z - \chi\phi\theta\beta \tanh\left(\frac{\alpha z}{2}\right) \tag{B.5}$$

The derivative with respect to z equals

$$h'(z) \quad = \quad \delta - \chi\phi\theta\beta\alpha\frac{1}{2}\left(1 - \tanh^2\left(\frac{\alpha z}{2}\right)\right). \tag{B.6}$$

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Table B.1: Quantitative Exercise Parameter Values

Parameter		Value	Source
Period length (years)		6	Avg. US debt maturity 2023
War probability (% p.a.)	ϕ	1.7%	Barro (2006)
Depreciation rate (% p.a.)	δ	8.0%	Cooley and Prescott (1995)
Discount factor	β	0.99	r-star from Laubach and Williams (2003)
Investor disaster risk aversion	ψ	4	Barro (2006)
Disaster economic contraction	b	35.0%	Barro (2006)
War discount factor	θ	5.60	Implied $\equiv (1-b)^{-\psi}$

From the proof of Proposition 2, it then follows that

$$\chi' = \frac{2\delta}{\phi\theta\alpha\beta}, \tag{B.7}$$

To derive the fragility threshold, note that the equilibrium mapping can be written as

$$z_{t-1} = H(z_t) \quad = \quad \frac{z_t - \chi\phi\theta\beta \tanh\left(\frac{\alpha z_t}{2}\right)}{1-\delta}. \tag{B.8}$$

The derivative is given by

$$H'(z_t) \quad = \quad \frac{1 - \frac{\chi\phi\theta\alpha\beta}{2}\left(1 - \tanh^2\left(\frac{\alpha z_t}{2}\right)\right)}{1-\delta} \tag{B.9}$$

The mapping H has a downward-sloping portion if and only if $\chi > \chi''$ where the fragility threshold is now given by

$$\chi'' \quad = \quad \frac{2}{\phi\theta\alpha\beta}. \tag{B.10}$$

B.3 Model Parameter Values

All parameter values are listed in Table B.1. We quantify the war discount rate following Barro (2006). Barro (2006) shows that in historical data, economic disasters typically have involved a drop in consumption of $b = 35\%$ and he shows that CRRA utility with a risk aversion parameter of $\psi = 4$ can explain the average equity premium observed in the data jointly with the historical frequency and severity of rare disasters. For us, this implies

$$\theta = (1-b)^{\psi}. \tag{B.11}$$

We use this link to solve for θ at different values of investor risk aversion ψ.

B.4 Effectiveness of military spending and back-of-the-envelope thresholds

Using commodity price shocks Federle, Rohner and Schularick (2024) estimate that an additional 10% of resources (in GDP units) increases government revenues by 5% and increases the probability of winning a war by 3.2%. Their sample consists of 135 countries since 1977. In the model

$$\frac{dw_i(m_i, m_{-i})}{dm_i} = \frac{\alpha}{4}\left(1 - \tanh^2\left(\frac{\alpha(m_1 - m_2)}{2}\right)\right) \leq \frac{\alpha}{4}. \tag{B.12}$$

Hence, if we have two contestants with relatively even winning probabilities then

$$\alpha = 4 \times \frac{dw_i(m_i, m_{-i})}{dm_i} = 4 \times \frac{3.2}{5} = 2.56. \tag{B.13}$$

If the contestants' winning probabilities are not even, this value needs to be interpreted as a lower bound for the effectiveness of military spending, α. With the other calibrated parameter values in natural (i.e. 6-year units), this gives

$$\chi' = \frac{2\delta}{\phi\theta\alpha\beta} = 56\%, \tag{B.14}$$

$$\chi'' = \frac{2}{\phi\theta\alpha\beta} = 152\%, \tag{B.15}$$

both expressed in units of GDP. To the extent that the back-of-the-envelope values for α represents a lower bound, the thresholds χ' and χ'' should be interpreted as upper bounds and hence as conservative values.

B.5 Symmetric countries plots

We plot the multiplicity and fragility thresholds relative to a baseline value, which allows us to inspect how the threshold functions vary without having to take a stance on the value of α, which is particularly hard to quantify. In particular, we plot χ' and χ'' scaled by the multiplicity threshold at some baseline parameter values $\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}$, i.e. we plot the quantities

$$\frac{\chi'(\phi, \theta, \beta, \delta, \alpha)}{\bar{\chi}'\left(\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}, \alpha\right)} = \frac{\delta}{\phi\theta\beta} / \left(\frac{\underline{\delta}}{\underline{\phi}\underline{\theta}\underline{\beta}}\right), \tag{B.16}$$

and

$$\frac{\chi''(\phi, \theta, \beta, \delta, \alpha)}{\bar{\chi}'\left(\underline{\phi}, \underline{\theta}, \underline{\beta}, \underline{\delta}, \alpha\right)} = \frac{1}{\phi\theta\beta} / \left(\frac{\underline{\delta}}{\underline{\phi}\underline{\theta}\underline{\beta}}\right) \tag{B.17}$$

To plot these thresholds against investor risk aversion ψ or the natural real rate on the x-axis, we use the link (B.11) and $\bar{r}_f = -\log\beta$.

B.6 Plots with asymmetric debt capacity χ

Now we consider the dynamic model with asymmetric ability to pledge future debt repayments, $\chi_1 > \chi_2$. For completenss, we also allow for $\tau \neq 1$, though we will show that this drops out for the plots of multiplicity/fragility thresholds relative to the equal debt capacity benchmark. The equilibrium dynamics for countries 1 and 2 are

$$\begin{aligned} m_{1t} &= \tau + (1-\delta)\, m_{1t-1} - \tau\chi_1(1-\beta) - \tau\chi_1\beta \times (\phi - \phi\theta F\left(\alpha\left(m_{1t} - m_{2t}\right)\right)) \\ m_{2t} &= \tau + (1-\delta)\, m_{2t-1} - \tau\chi_2(1-\beta) - \tau\chi_2\beta \times (\phi - \phi\theta\left(1 - F\left(\alpha\left(m_{1t} - m_{2t}\right)\right)\right)) \end{aligned}$$

Taking the difference between countries 1 and 2 yields

$$\mu_t - (1-\delta)\,\mu_{t-1} = (-\phi\beta - (1-\beta))\,\tau\,(\chi_1 - \chi_2) + \tau\chi_1 \times \phi\theta\beta F(\alpha\mu_t) - \tau\chi_2 \times \phi\theta\beta\,(1 - F(\alpha\mu_t))\,,$$
$$= \left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\,(\chi_1 - \chi_2) + \tau\frac{\chi_1 + \chi_2}{2} \times \phi\theta\beta(2F(\alpha\mu_t) - 1). \tag{B.18}$$

A steady-state must be a zero of the function

$$h(z) = \frac{\delta}{\alpha}\alpha z - \frac{\delta}{\alpha} \times \frac{\alpha}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\,(\chi_1 - \chi_2) - \tau\frac{\chi_1+\chi_2}{2} \times \phi\theta\beta\tanh\left(\frac{\alpha z}{2}\right). \tag{B.19}$$

Comparing to the proof of Proposition 2 , there is a unique steady state if and only if

$$\tilde{h}\left(\frac{\tau}{\delta}\frac{\chi_1+\chi_2}{2} \times \phi\theta\beta\alpha\right) < \frac{\alpha}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)\tau\,(\chi_1 - \chi_2)\,. \tag{B.20}$$

The multiplicity threshold for χ_1 at a given ratio χ_2/χ_1 then must satisfy

$$\tilde{h}\left(\chi_1\alpha\frac{\tau}{\delta}\frac{1+\chi_2/\chi_1}{2}\phi\theta\beta\right) - \chi_1\alpha\frac{\tau}{\delta}\left(-\phi\beta - (1-\beta) + \frac{\phi\theta\beta}{2}\right)(1 - \chi_2/\chi_1) = 0. \tag{B.21}$$

We plot the multiplicity threshold scaled by the baseline with equal debt capacity, i.e.

$$\frac{\chi_1'}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} = \frac{\alpha\tau\chi_1'}{\frac{2\delta}{\phi\theta\beta}}. \tag{B.22}$$

We can find the product $\alpha\tau\chi_1'$ numerically as the zero of (B.21). Note that the solution for the product $\alpha\tau\chi_1'$ is independent of α and τ, so the ratio (B.22) is also independent of α and τ. We verify numerically that there is only one zero, so this zero gives the unique threshold such that there are multiple steady states when χ_1 exceeds the threshold.

Following the proof of Proposition 3, the fragility threshold for χ_1 for a given ratio χ_2/χ_1 is given by

$$\chi_1'' = \frac{2}{\tau\phi\theta\beta\alpha}\frac{1}{\frac{1}{2}\left(1 + \frac{\chi_2}{\chi_1}\right)}, \tag{B.23}$$

and we plot this fragility threshold scaled by the baseline multiplicity threshold:

$$\frac{\chi_1''}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} = \frac{1}{\frac{\delta}{2}\left(1 + \frac{\chi_2}{\chi_1}\right)}. \tag{B.24}$$

Note that this second ratio is also independent of α and τ, so we do not need to take a stance on these quantities to understand how the multiplicity and fragility thresholds vary compared to the benchmark where both countries have equal access to international financial markets.

B.7 Plots with asymmetric country size τ

Now, consider the case with $\tau_1 > \tau_2$ and equal debt capacity $\chi_1 = \chi_2 = \chi$. The dynamic equation for country i's budget constraint becomes

$$m_{it} = \tau_i + (1-\delta)\, m_{it-1} + (\beta - 1)\tau_i - \tau_i \chi \times \phi\beta \left(1 - \theta w_i(m_{it}, m_{-it})\right)$$

Taking the difference between countries 1 and 2 gives

$$\begin{aligned}\mu_t - (1-\delta)\,\mu_{t-1} &= (\tau_1 - \tau_2)\left(1 - \chi(1-\beta) - \chi \times \phi\beta\right) + \tau_1 \chi \times \phi\theta\beta F(\alpha\mu_t) - \tau_2 \chi \times \phi\theta\beta \left(1 - F(\alpha\mu_t)\right), \\ &= (\tau_1 - \tau_2)\left(1 - \chi(1-\beta) + \chi \times \phi\beta\left(\frac{\theta}{2} - 1\right)\right) + \frac{\tau_1 + \tau_2}{2}\chi \times \phi\theta\beta \tanh\left(\frac{\alpha\mu_t}{2}\right) \quad (B.25)\end{aligned}$$

Here, we have used that $2F(\alpha\mu_t) - 1 = \tanh\left(\frac{\alpha\mu_t}{2}\right)$. Assume that the war discount rate θ is high enough that the intercept in (B.25) is positive. A steady state is a zero of the following function

$$h(z) = \frac{\delta}{\alpha}\left(\alpha z - \frac{\alpha}{\delta}(\tau_1 - \tau_2)\left(1 - \chi(1-\beta) + \chi \times \phi\beta\left(\frac{\theta}{2} - 1\right)\right)\right) - \frac{\tau_1 + \tau_2}{2}\chi \times \phi\theta\beta \tanh\left(\frac{\alpha z}{2}\right).$$

Comparing to equation (A.12) in the proof of Proposition 2 shows that there exists a unique steady state with country 1 dominating if and only if $\frac{\alpha}{\delta}\frac{\tau_1+\tau_2}{2}\chi \times \phi\theta\beta < 2$ or if

$$\tilde{h}\left(\frac{\frac{\tau_1+\tau_2}{2}\chi \times \phi\theta\beta\alpha}{\delta}\right) < \frac{\alpha}{\delta}(\tau_1 - \tau_2)\left(1 - \chi(1-\beta) + \chi \times \phi\beta\left(\frac{\theta}{2} - 1\right)\right). \quad (B.26)$$

The right-hand-side in (B.26) does not scale with χ, so the solution for $\alpha\chi$ increases with α. We plot the zero of (B.26) scaled by the multiplicity threshold when both countries have equal tax revenue, i.e.

$$\frac{\chi'(\tau_1 \neq \tau_2)}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} \quad (B.27)$$

The fragility threshold is given by

$$\chi'' = \frac{2}{\phi\beta\theta\alpha\frac{\tau_1+\tau_2}{2}}, \quad (B.28)$$

showing that it decreases as τ_1 increases. We plot

$$\frac{\chi''(\tau_1 \neq \tau_2)}{\frac{2\delta}{\alpha\tau\phi\theta\beta}} \quad (B.29)$$

C Alternative Contest Functions

While our main results rely on the difference contest function of Hirshleifer (1989), the main results in the on complementarity and multiplicity are robust to assuming one of the other most popular functional forms, i.e. a ratio contest function or even a linear contest function (though in that case military capacity needs to be bounded). We show robustness in the two-period model.

C.1 Ratio Contest Function

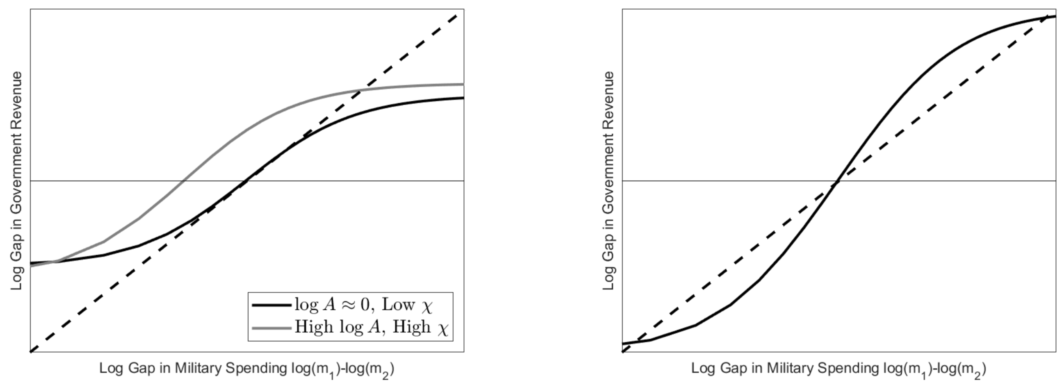
We start with a ratio contest function, where we now assume

$$w_1(m_1, m_2) = F\left(\frac{A \times m_1}{m_2}\right) = \frac{\left(\frac{A \times m_1}{m_2}\right)^{\kappa}}{1 + \left(\frac{A \times m_1}{m_2}\right)^{\kappa}}, \tag{C.1}$$

for some constant κ.

Figure C.1: Equilibria in Two-Period Model with Ratio Contest Function

Panel A: Unique Equilibrium **Panel B:** Multiple Equilibria



This figure is analogous to Figure 4 in the main paper, but it uses a ratio contest function (C.1). The difference in log military capacity $\log m_1 - \log m_2$ is depicted on the x-axis, and the gap in log government revenue $\log(\tau + q_1 b_1) - \log(\tau + q_2 b_2)$ is depicted on the y-axis. An equilibrium is defined by the intersection of the solid line and the dotted 45-degree line. Panel A depicts two examples of unique equilibria. Panel B depicts an example of multiple equilibria. In Panel A, the solid black line represents the gap in log government revenue under a low exogenous military advantage (i.e. $\log A \approx 0$). Debt capacity is sufficiently low that the slope of the curve is always less than one. The solid gray line represents the gap in government revenue under a high exogenous military advantage $\log A >> 0$ and high debt capacity. The maximum slope of this curve exceeds one, but the exogenous military advantage A is large enough to ensure equilibrium uniqueness. Panel B depicts the gap in log government revenue under a small exogenous military advantage (i.e. $\log A \approx 0$) and high debt capacity, so that the maximum slope of this curve exceeds one and there are multiple equilibria.

We show numerically that the results from the main paper carry over. Starting from the equilibrium military investment

$$m_i \quad = \quad \tau + \tau\chi \times (1 - \phi + \phi\theta w_i(m_i, m_{-i})) \,. \tag{C.2}$$

Taking the difference in log military expenditures gives

$$\begin{aligned} \log m_1 - \log m_2 \quad = \quad & \log\left(\tau + \tau\chi \times (1 - \phi + \phi\theta F\left(\log(A) + \log(m_1) - \log(m_2)\right)\right)) \\ & - \log\left(\tau + \tau\chi \times (1 - \phi + \phi\theta\left(1 - F\left(\log(A) + \log(m_1) - \log(m_2)\right)\right)\right)) \end{aligned} \tag{C.3}$$

where with an abuse of notation, we have written the contest function F a a function of $\log A + \log m_1 - \log m_2$. Equation (C.3) is an equilibrium condition for the log difference in military capacity. We plot this equilibrium condition numerically in Figure C.1. It shows that, as in our baseline case, there are two ways to obtain a unique equilibrium. Either if the slope of the revenue curve is low (such as when χ is low) or when one country has a strong exogenous advantage ($\log A \gg 0$). Panel B shows that multiple equilibria obtain when the exogenous advantage is small (now $\log A \approx 0$) and when the slope of the revenue curve is high (χ above a threshold). The similarity with Figure 4 hence shows that the features of the two-period model are very similar when we assume a ratio contest function, except that the level of military capacity now needs to be replaced with logs.

C.2 Linear Contest Function

We now replace equation (1) in the main paper with the linear contest function

$$w_1(m_1, m_2) = \frac{1}{2} + \kappa(A + m_1 - m_2), \text{ s.t. } m_i \leq \bar{m}. \tag{C.4}$$

The bounds on military capacity, m_i, are necessary to ensure that the probability of winning always remains between zero and one. Intuitively, they reflect that it becomes infinitely costly to invest enough to push the probability of winning above a given bound. We assume $\bar{m} > \tau$ to avoid the trivial case where countries reach the upper bound without borrowing.

Proposition 1' *There exists a threshold $\chi' > 0$ such that if $\chi > \chi'$ there exists an equilibrium where country 2 dominates.*

Proof: Equilibrium military investment of country i satisfies the complementary slackness conditions:

$$m_i = \tau + \tau\chi \times (1 - \phi + \phi\theta w_1(m_1, m_2)) \text{ if } m_i < \bar{m} \tag{C.5}$$

$$m_i < \tau + \tau\chi \times (1 - \phi + \phi\theta w_1(m_1, m_2)) \text{ if } m_i = \bar{m}. \tag{C.6}$$

An equilibrium that is interior for both countries hence must satisfy

$$m_1 - m_2 = \tau\chi \times \phi\theta \left(2\kappa \left(A + m_1 - m_2\right)\right), \tag{C.7}$$

$$m_1 - m_2 = \frac{\tau\chi \times \phi\theta \times 2\kappa A}{1 - \tau\chi \times \phi\theta \times 2\kappa}. \tag{C.8}$$

Substituting back into (C.5) gives

$$m_1 = \tau + \tau\chi \times \left(1 - \phi + \phi\theta \left(\frac{1}{2} + \kappa \frac{A}{1 - \tau\chi \times \phi\theta \times 2\kappa}\right)\right), \tag{C.9}$$

$$m_2 = \tau + \tau\chi \times \left(1 - \phi + \phi\theta \left(\frac{1}{2} - \kappa \frac{A}{1 - \tau\chi \times \phi\theta \times 2\kappa}\right)\right). \tag{C.10}$$

Hence, this interior equilibrium exists if (C.9) is less than $\bar{m}$. It is also clear that in such an interior equilibrium country 1 is dominates, provided that $A > 0$.

Next, we ask under which conditions there exists an equilibrium where $m_2 = \bar{m}$ and $m_1 < \bar{m}$, i.e. there exists an equilibrium where country 2 dominates. The complementary slackness conditions give that such an equilibrium must satisfy

$$m_1 = \tau + \tau\chi \times \left(1 - \phi + \phi\theta \left(\frac{1}{2} + \kappa \left(A + m_1 - \bar{m}\right)\right)\right) \tag{C.11}$$

and

$$\bar{m} < \tau + \tau\chi \times \left(1 - \phi + \phi\theta \left(\frac{1}{2} - \kappa \left(A + m_1 - \bar{m}\right)\right)\right). \tag{C.12}$$

Because the probability of winning is bounded between zero and one, condition (C.12) is satisfied if $\chi > \frac{\bar{m}-\tau}{\tau(1-\phi)}$. Conversely, because of the assumption that $\bar{m} > \tau$, condition (C.12) is violated as $\chi \to 0$. This proves the claim.

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