

Global Hegemony and Exorbitant Privilege *

Carolyn E. Pflueger

Pierre Yared

June 1, 2025

Abstract

We present a dynamic two-country model in which military spending, geopolitical dominance, and government bond prices are jointly determined. The model reflects three facts: hegemon enjoy a funding advantage, this advantage rises with geopolitical tensions, and war losers devalue their debts more. In the model, greater bond revenue enables military investment, in turn increasing the safety value of bonds to international investors. Debt capacity strengthens the hegemon's military and financial advantage but introduces steady-state multiplicity and fragility. With intermediate capacity, initial conditions determine the hegemon. However, with high capacity, self-fulfilling bond market expectations can trigger hegemonic transitions and geopolitical fragility.

Keywords: Fiscal Policy, International Conflicts, International Finance, National Security and War, Safe Assets

JEL Classifications: E62, F51, F65, H56

*We would like to thank Fernando Broner, Christopher Clayton, David Hirshleifer, Rishabh Kirpalani, Dimitris Papanikolaou, Ken Rogoff, Jesse Schreger, Christoph Trebesch and seminar participants at the University of Zurich, the Federal Reserve, Purdue University, Northwestern Kellogg, the 3rd Kiel-CEPR Conference on Geoeconomics, the 2024 BFI International Macro-Finance Conference, the Spring 2025 Macro-Finance Society meeting, and the Stanford SITE 2024 session on fiscal sustainability for comments, and Kyler Kirk for excellent research assistance. Pflueger: University of Chicago, Harris School of Public Policy and NBER, cpflueger@uchicago.edu; Yared: Columbia Business School and NBER, pyared@columbia.edu. Yared completed his work on this project prior to joining the Council of Economic Advisers in the Executive Office of the President of the United States. The views expressed do not reflect those of the CEA.

1 Introduction

A government’s ability to borrow to finance military spending has historically played a critical role in military success. During the Napoleonic Wars, for example, Great Britain’s credibility with lenders across Europe allowed for significant debt-financing, and delivered a military advantage over France, leading to Great Britain’s subsequent military and financial dominance throughout the 19th century (Kennedy (1987), Bordo and White (1991), Sargent and Velde (1995)).¹

Much like Great Britain in the nineteenth century, the U.S. today serves as the global military hegemon and it also benefits from an “exorbitant privilege” enjoying low borrowing rates in international bond markets. Bond markets are also clearly of first-order importance for policy decision-making today as evidenced by the threats from recurrent “bond market vigilantes”; and this is especially pertinent for military spending, which is the largest component of marginal, discretionary spending.² In a world with renewed international military tensions and, in particular, a growing economic and military rivalry between the U.S. and China, this raises the question whether the bond markets that have fueled U.S. military investment and spending for so long might themselves turn into a source of geopolitical fragility.

This paper examines the link between government financing and geopolitical rivalry in a simple dynamic two-country model with international bond markets. We ask the following three questions: How does the debt capacity of international bond markets impact the military balance between countries? How does the military balance affect the financing advantage of the dominant country? And how do hegemonic transitions take place?

In the first part of the paper, we document three empirical observations to motivate the model. First, using evidence on historical government borrowing rates around the time of hegemonic transitions, we document that military hegemons tend to borrow at lower interest rates; moreover, changes in the identity of the country that enjoys the borrowing advantage have historically coincided with shifts in military standing. Second, we document that the interest spread for different countries relative to the global hegemon rises when geopolitical tensions increase. Third, we present evidence that in the aftermath of realized geopolitical conflict, losers experience greater inflation and debt devaluation compared to countries that emerge relatively unscathed.

In the second part of the paper, we construct a simple dynamic two-country model consistent with these three facts in order to address our motivating questions. In the model, military strength is a function of endogenous accumulated military spending, which we refer to as military capacity, and exogenous factors (such as geography or technology). We allow one country to have an exogenous advantage over the other, but otherwise assume symmetry across countries. Countries decide how much to spend on defense versus other public goods, how much to borrow, and whether to default. Countries’ budget constraints—and hence ability to invest in military capacity—are a function of

¹Similar dynamics applied in the U.S. Civil War, with the Union benefiting from greater access to international capital markets than the Confederacy, which faced challenges guaranteeing its cotton-backed bonds in the presence of the Union’s naval blockade (McCandless (1996), Ferguson (2008))

²The Congressional Budget Office discretionary spending infographic shows that the majority of 2023 discretionary spending was military, including veteran’s benefits and international affairs <https://www.cbo.gov/publication/59729>.

exogenous tax revenue plus revenue from borrowing in international bond markets. We assume that losing a war induces a direct cost, which we take to be large. As is common in the sovereign debt literature, we model the cost of default as an exogenous proportional endowment cost, which determines a country’s ability to pledge future taxes to bond investors. We refer to this ability to commit to repay—which in equilibrium equals international investors’ global liquidity demand—as “debt capacity” for short. Consistent with our third motivating empirical fact, we assume that military defeat results in the complete destruction of a country’s endowment, implying that a defeated country optimally chooses not to repay its debt.

Bond market revenue and military capacity in the model are interlinked. The likelihood that a country wins a war is assumed to be a function of relative military strength, modeled as a contest function ([Hirshleifer \(1989\)](#)). War occurs exogenously with some positive probability in every period. Because international investors are assumed to value financial assets that pay out in wars and other disasters ([Rietz \(1988\)](#), [Barro \(2006\)](#)), bonds of the militarily stronger country are valued more highly for their safety, resulting in a feedback loop between military capacity and the ability to raise revenue in the bond market.

The model generates strikingly different results for different levels of debt capacity, illustrating the relevance of international financial markets for military and borrowing cost dynamics. First, if debt capacity is low, there is a unique steady state conditional on remaining in a state of peace, with the exogenously stronger country winning (in expectation) if war takes place. The weaker country loses (in expectation) and therefore is *ex ante* more likely to default when war is realized. Thus, and in line with our first motivating empirical fact, the stronger country benefits from a lower funding cost. Moreover, and in line with our second motivating empirical fact, an increase in the likelihood of war due to higher geopolitical tensions raises the interest spread between the two countries, since greater war risk raises the defeat and default risk for the weaker country relative to the stronger country.

An important feature of this steady state is the complementarity between military strength and funding advantage. As debt capacity increases for both countries, the funding advantage of the stronger country also increases, translating into greater relative military spending and a larger probability of winning. Analogously, as the exogenous military advantage of the stronger country increases, so does its funding advantage, which amplifies the military advantage by facilitating an increase in endogenous military spending.

Second, if the debt capacity is intermediate or high, there are multiple steady states conditional on remaining in a state of peace. While the steady state in which the stronger country wins the war is preserved, a second steady state emerges in which the weaker country wins (in expectation). This second steady state is sustained by bond market participants’ anticipation that the exogenously weaker country will invest enough in the military to overcome the stronger country’s exogenous military advantage. That belief underpins a lower funding cost for the exogenously weaker country, which is now less likely to default; this, in turn supports a larger military investment by the weaker country facilitated by the country’s debt capacity. Other factors beyond debt capacity also support

multiplicity of steady states. Multiplicity is more likely if the exogenous difference in military capability across the two countries is small so that the weaker country can more feasibly dominate the stronger country. It is also more likely if the depreciation rate on military capital stock is low, so that the exogenously weaker country can build up enough capacity over time to overwhelm the stronger country. In addition, a higher likelihood of war or a higher war risk premium enable multiplicity since these increase investors' valuation for safe bonds with relatively higher payouts in the particularly valuable war state.

Our third result considers the dynamic evolution of the dominant country's military and funding advantage, and we show that this differs for intermediate vs. high debt capacity. At intermediate debt capacity, the military and financial advantages are built up slowly over time, and initial conditions determine the eventual steady state. By contrast, at high debt capacity, bond market expectations can be self-fulfilling and lead to sudden non-monotonic dynamics. Conditional on remaining in a state of peace, there always exists a convergent monotonic path towards the steady state for any debt capacity. If debt capacity is intermediate, then this path is unique. An implication of this result is a form of *geopolitical hysteresis* that emerges for intermediate debt capacity: the initial relative level of military power determines which country will ultimately dominate militarily and financially, and the dominant country's military and funding cost advantages grow over time. Only the realization of the war state can change the balance of power and is required for a hegemonic transition. Following a war-induced reset, the new hegemon consolidates its military and financial advantages over time, eventually converging to the new steady state.

By contrast, if debt capacity is high, then the convergent path is not unique, defining a “zone of fragility” where initial conditions do not necessarily predict the ultimate steady state and non-monotonic dynamics are possible. We term this result *geopolitical fragility*. In this case, hegemonic transitions can occur even in the absence of war, coordinated by expectations in bond markets. Intuitively, at high debt capacity, changes in bond prices impact countries' budget constraints so strongly that expectations can become self-fulfilling. Geopolitical fragility is more likely the higher the likelihood of war and the higher the investor war risk premium. The two countries can subsequently transition out of the zone of fragility and converge towards a new steady state, with the military and financial dominance of the new hegemon being solidified over time. Overall, the dynamic analysis shows that the dominant country's military and funding advantages can be ephemeral when debt capacity is high.

A key insight from the intermediate and high debt capacity cases is that factors that enhance the complementarity between geopolitical and financial dominance—higher debt capacity, higher war probability, and higher war risk premium—also entail costs for the stronger country. They introduce steady-state multiplicity with the possibility that the stronger country is overwhelmed by the weaker country. They also introduce fragility in the sense that the equilibrium may no longer exhibit monotonic convergence, and may switch to a new path converging to a different hegemon.

In an extension of our environment with asymmetric countries, we find that financial development that raises the debt capacity of the weaker country or impairs debt capacity of the stronger

country raises the likelihood of multiplicity; in contrast, a greater gap in debt capacity favors uniqueness and dominance of the country with higher debt capacity. This extension provides a useful lens for interpreting the dominance of Great Britain over France during the Napoleonic Wars. It also implies that any successful attempt to internationalize China’s currency and raise China’s debt capacity relative to the U.S. is likely to matter for U.S. national security. The same is true for any U.S. policy decisions—such as a technical default on U.S. Treasury bonds due to a failure to raise the debt ceiling—that erode its own debt capacity.

This paper builds on the literature studying rationalist models of war going back to the work of [Fearon \(1995\)](#) and [Powell \(1993, 1999\)](#).³ Relative to this literature, which has often emphasized asymmetric information, we abstract away from decisions to go to war. Instead, we focus on the role of international debt markets, which endogenizes the resource constraint of a country choosing armaments and introduces multiplicity and fragility.⁴

Our paper is complementary with the recent growing literature that explores the effects of geopolitical risk and global hegemony on globalization (e.g. [Clayton, Maggiori and Schreger \(2023\)](#)), building on the landmark contributions of [Hirschman \(1945, 1958\)](#).⁵ The focus of this literature is on trade volumes, trade networks, and trade policy, whereas our focus is on the interaction between global debt markets and military spending.

This paper also contributes to the political economy literature on government debt. This literature emphasizes the role that political incentives play in driving government debt dynamics.⁶ Most related is the work that considers these incentives and coordination in open economies with multiple countries (e.g., [Chari and Kehoe \(2007\)](#)), [Azzimonti, de Francisco and Quadrini \(2014\)](#), [Aguiar et al. \(2015\)](#), [Halac and Yared \(2018\)](#), [Dovis and Kirpalani \(2020\)](#), [Fornaro and Grosse-Steffen \(2025\)](#)). We contribute to this literature by considering how defense spending and the prospect of military conflict influences bond prices and government policies.⁷

Finally, there is a large literature that has studied the “exorbitant privilege” of the U.S., referring to its ability to borrow large amounts in its own currency at relatively low rates ([Eichengreen \(2011\)](#), [Gourinchas and Rey \(2022\)](#), [Meissner and Taylor \(2006\)](#), [Atkeson, Heathcote and Perri \(2022\)](#), [Choi, Kirpalani and Perez \(2024\)](#)). More specifically, we relate to work studying the rise and fall of bond market dominance and its determinants. The prior literature has studied the link from bond

³Among others, see also [Skaperdas \(1992\)](#), [Baliga and Sjöström \(2004\)](#), [Acemoglu et al. \(2012\)](#), [Acemoglu and Wolitzky \(2011\)](#), [Padró i Miquel and Yared \(2012\)](#), and [König et al. \(2017\)](#).

⁴[Gennaioli and Voth \(2015\)](#) also endogenize the resource constraint of governments by considering the decision to engage in state capacity building, but they do not consider issues of debt markets, multiplicity, or hegemonic transitions.

⁵In addition, see [Maggi \(2016\)](#), [Becko and O’Connor \(2024\)](#), [Broner et al. \(2024\)](#), [Liu and Yang \(2024\)](#), [Fernández-Villaverde, Mineyama and Song \(2024\)](#), and [Thoenig \(2024\)](#). See [Mohr and Trebesch \(2024\)](#) for a review.

⁶There is a large literature dating back to [Persson and Svensson \(1989\)](#) and [Alesina and Tabellini \(1990\)](#) that considers these issues, and a more recent literature on the pricing of political uncertainty in financial markets ([Pastor and Veronesi \(2012\)](#), [Pastor and Veronesi \(2020\)](#), [Kelly, Pastor and Veronesi \(2016\)](#), [Koijen, Philipson and Uhlig \(2016\)](#)). See [Alesina and Passalacqua \(2016\)](#) and [Yared \(2019\)](#) for surveys of the literature.

⁷Our work also relates to the literature on multiplicity in sovereign default models dating back to [Cole and Kehoe \(2000\)](#). As in this literature, here self-fulfilling investor expectations change default risk. However, in our framework self-fulfilling expectations come at the expense of another borrower and occur through endogenous changes in geopolitical risk.

market dominance to trade invoicing (Gopinath and Stein, 2021; Chahrour and Valchev, 2022), market power in the issuance of safe debt (Farhi and Maggiori, 2018), risk premia due to market size and credibility (Hassan (2013), Du, Pflueger and Schreger (2020), Clayton et al. (2025)), and fiscal position (Chen et al., 2025). Our analysis is complementary to work that emphasizes the role of market depth and liquidity for bond market dominance (He, Krishnamurthy and Milbradt (2019), Coppola, Krishnamurthy and Xu (2023)) since our mechanism builds on the interaction between these financial market features and geopolitical strength. We contribute to this literature by theoretically modeling the link between military and financial dominance in a framework that elucidates issues of multiplicity, fragility, and hegemonic transitions.

2 Motivating Empirical Facts

This section provides empirical evidence motivating our theoretical model. First, militarily stronger countries enjoy a borrowing cost advantage, and changes in the military balance have historically coincided with changes in who enjoys the lowest borrowing costs; second, this funding advantage increases with the ex-ante probability of a military conflict or geopolitical tensions; and third, the winners from wars have historically devalued their debts less than losers. While these facts build on the large empirical literatures on historical borrowing costs, geopolitical tensions, and rare disasters, we emphasize their connection motivating our model.

2.1 Funding Advantage Switches with Military Strength

Militarily stronger countries have historically experienced a lower cost of funding, and this fact can be illustrated by considering the interest rates faced by governments around the time of hegemonic transitions. Such a transition occurred from Great Britain to the U.S. in the period spanning World War I and World War II, a period during which the British Empire went from being the dominant military power worldwide to a secondary power faced with losing its colonies. This shift in military power coincided with a shift in the financial center from London to New York, and a flip in the ordering of U.S. and U.K. borrowing costs, as shown in Figure 1, Panel A.⁸ Remarkably, the shift from a U.K. borrowing advantage pre-WWI to a U.S. borrowing disadvantage compared to the U.S. was long-lived and persisted after the cessation of hostilities well into the second half of the 20th century.⁹

An earlier military and financial transition occurred at the end of the eighteenth century. Prior to this period, the Netherlands was a dominant global trading and naval power, as exemplified

⁸We plot U.S. and U.K. 10-year government bond yields from 1870 until 1970. During this period, both countries were mostly on the gold standard or their exchange rates were fixed by Bretton Woods, so no inflation adjustment is needed.

⁹Schmelzing (2020) and Ilzetzi, Reinhart and Rogoff (2022) have noted that on the eve of World War I, German imperial bonds experienced a brief 5-year period in which they dominated those of Great Britain, suggesting that Germany may have been perceived as the military hegemon during the great naval arms race with Britain prior to its defeat in the calamity of World War I. This is consistent with our model, where the dominant country is the one that is expected to win, though it may still lose depending on the state of the world that is realized.

by the Dutch East India Company and significant overseas territories, such as “New Amsterdam” (now New York) (Schama (1988)). The Dutch navy posed a formidable challenge to the British throughout the 16th, 17th, and 18th centuries, with an unequaled borrowing capacity that was the envy of every other naval administration. It even invaded Great Britain during the Glorious Revolution of 1688 with 463 ships and 40,000 men (about twice the size of the Spanish armada a century earlier) (Rodger, 2006, pp.137-142). After the Dutch defeat in the Anglo-Dutch War of 1780, invasion by Napoleonic armies in 1795, and the bankruptcy of the Dutch East India Company in 1796, the financial center of the world moved from Amsterdam to London. Figure 1 Panel B shows that the Netherlands’ funding cost advantage became a funding cost disadvantage compared to Britain at this time, and did not recover.¹⁰ The British emerged as clearly the strongest military and financial power from the military events of the early 1800s, and enjoyed a persistent funding cost advantage thereafter.

2.2 Geopolitical Risk and Funding Advantage

A large literature has documented that increases in geopolitical tensions tend to reduce the borrowing costs of militarily stronger countries relative to militarily weaker countries. Compiling extensive data from asset prices in wars and other rare disasters, Barro (2006) shows that U.S. borrowing costs generally fall during wars, and concludes that U.S. bonds are perceived as safe in such calamities. For the period when the U.K. was the militarily strongest country, Ferguson (2006), documents that British bond prices barely fluctuated in response to geopolitical tensions while bond prices of all other European countries tended to drop, i.e. the hegemon’s (in this case the U.K.) funding advantage widened with geopolitical tensions. More recently, Hirshleifer, Mai and Pukthuanthong (2025) provide complementary evidence that risk premia in U.S. Treasury bonds decline with global conflict, and Rigobon and Sack (2005) show that U.S. bond risk premia declined around high-frequency events associated with the 2003 Iraq war.

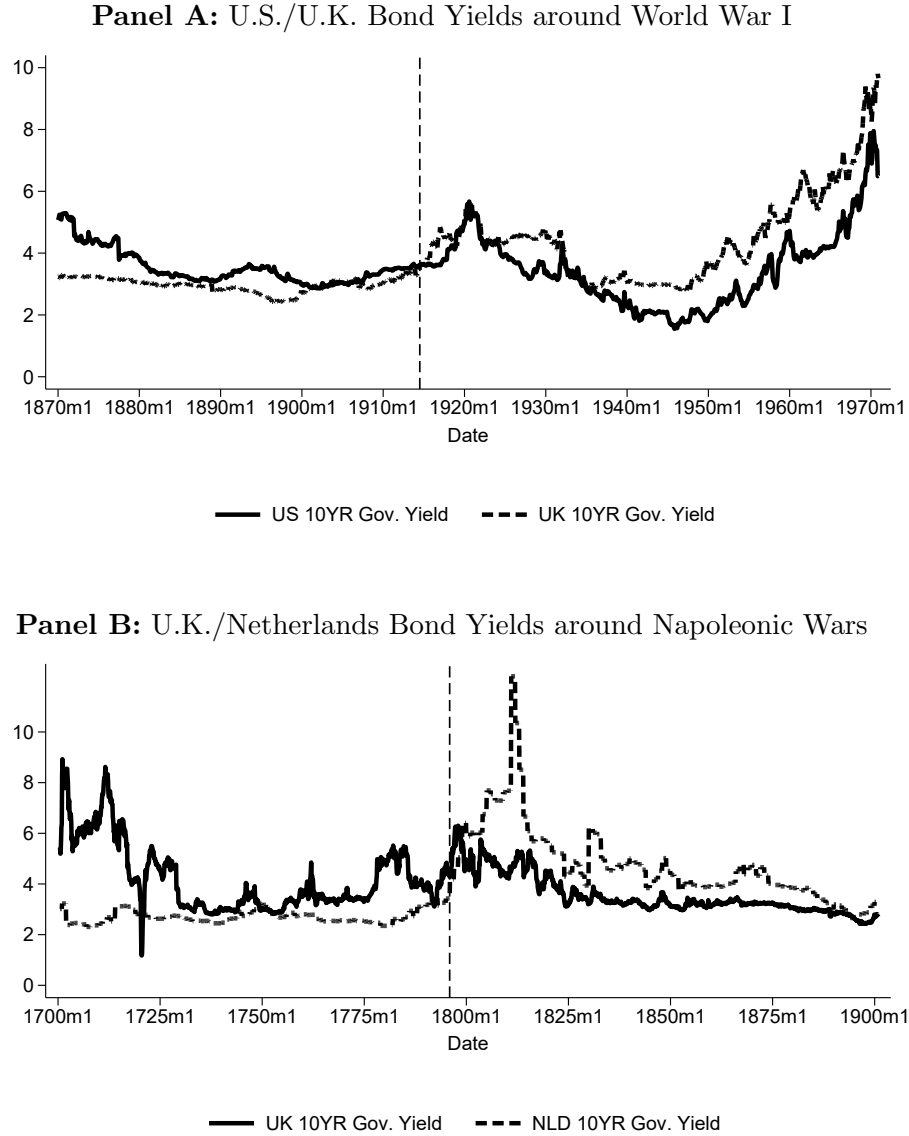
Figure 2 illustrates this association between the funding advantage with geopolitical tensions for the U.S. over the period starting in 1980.¹¹ The U.S. funding advantage is computed as the difference between an average of developed country three-year bond yields minus the U.S. three-year government bond yields relative to a 10-year moving average, where a higher value should be interpreted as a higher U.S. funding advantage. We see that the U.S. funding advantage was 45% correlated with a newspaper-based index of U.S. geopolitical risk index from Caldara and Iacoviello (2022).¹² While this index of geopolitical tensions—like most indices based on newspaper articles—is somewhat volatile, the relationship is strong. In particular, the U.S. borrowing cost

¹⁰This figure is similar to Figure 11 of Chen et al. (2025). Different from them, we focus on the coincidence of major shifts in military power with a switch in relative borrowing costs.

¹¹We start in 1980 because this is when bond yields for a broad cross-section of developed countries are available for comparability and when inflation was no longer a major concern in most developed countries.

¹²The correlation is notably stronger for the U.S. than for a spread of developed country yields over Japanese yields, suggesting that this correlation reflects U.S. military strength rather than flight to safe assets. The relationship in Figure 2 is statistically significant and of a similar magnitude without detrending, and when controlling for U.S. recessions or U.S. GDP growth relative to other developed countries.

Figure 1: Changing Military Dominance and Bond Yields in History

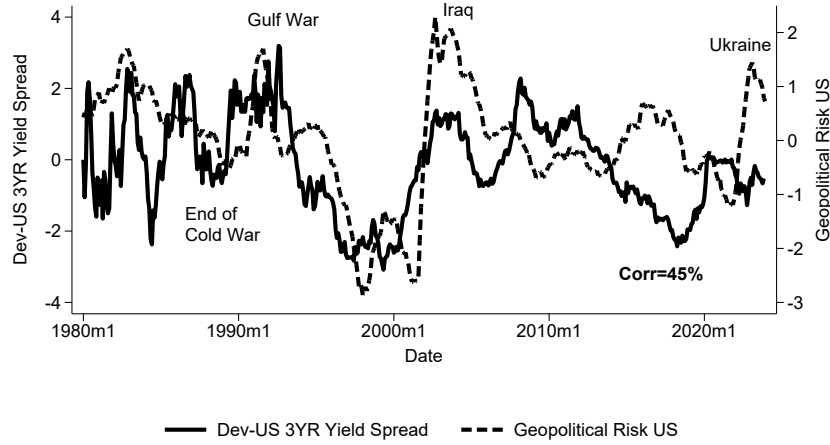


Panel A plots the 10-year government bond yields for the U.S. and the U.K. from January 1870 until December 1970 (end of Bretton Woods). A vertical line indicates the start of World War I (July 1914). Panel B plots the 10-year government bond yields for the U.K. and Netherlands from 1700 (when U.K. yields become available) until 1900. The vertical line indicates 1795, when the Netherlands was invaded by Napoleon. U.S. and U.K. bond yields are from Global Financial Data. Netherlands bond yields are from [Korevaar, Francke and Eichholtz \(2020\)](#). The Netherlands yield for the 1700s uses data for the Province of Holland.

advantage declined along with geopolitical tensions after the end of the Cold War during the 1990s, subsequently increased after 9/11 and tensions over the Iraq war, and most recently spiked with the invasion of Ukraine.¹³

¹³A deviation of the geopolitical tensions index and the U.S. funding advantage occurred around 2016, when

Figure 2: Developed Country-U.S. Borrowing Costs vs. Geopolitical Risk



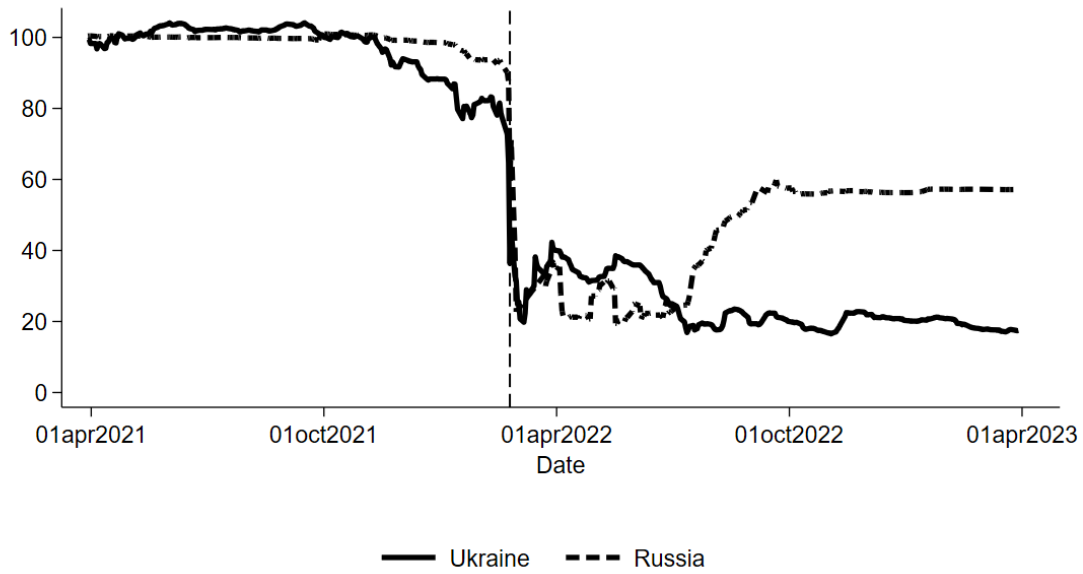
This figure plots the spread between an equal-weighted average three-year government yield from five developed countries (Australia, France, Germany, Japan, U.K.) minus the three-year U.S. government bond yield against the historical geopolitical risk index for the U.S (in annualized %). (GPHRC USA) from [Caldara and Iacoviello \(2022\)](#). The yield spread is stochastically detrended by subtracting a 10-year moving average, with all yield data from Global Financial Data. The German three-year government bond yield is extended with Bloomberg data after 2021. We take the log of the geopolitical risk index and a 12-month moving average to reduce the effect of outliers. Our sample is monthly and starts in January 1980 (when French government bond yields become available) and ends in November 2023. Geopolitical risk is standardized to have a mean of zero and a standard deviation of 1.

Figure 3 complements these broad patterns of geopolitical tensions with high-frequency evidence around the surprise invasion of Ukraine by Russia on February 24, 2022. Panel A of Figure 3 shows that Ukrainian and Russian government bond prices (which are inversely related to yields) fell precipitously around the invasion, suggesting that investors priced in a much higher probability of not getting repaid in full. The fall in Ukrainian bond prices was even larger than that of Russian bond prices, particularly after liquidity for Russian bonds improved in the second half of 2022, suggesting that the consequences of the war for Ukraine were priced to be particularly severe. At the same time, despite sanctions and the precipitous drop in bond prices, Russia was able to continue to finance its war efforts with commodity exports ([Gorodnichenko, Korhonen and Ribakova \(2024\)](#)).

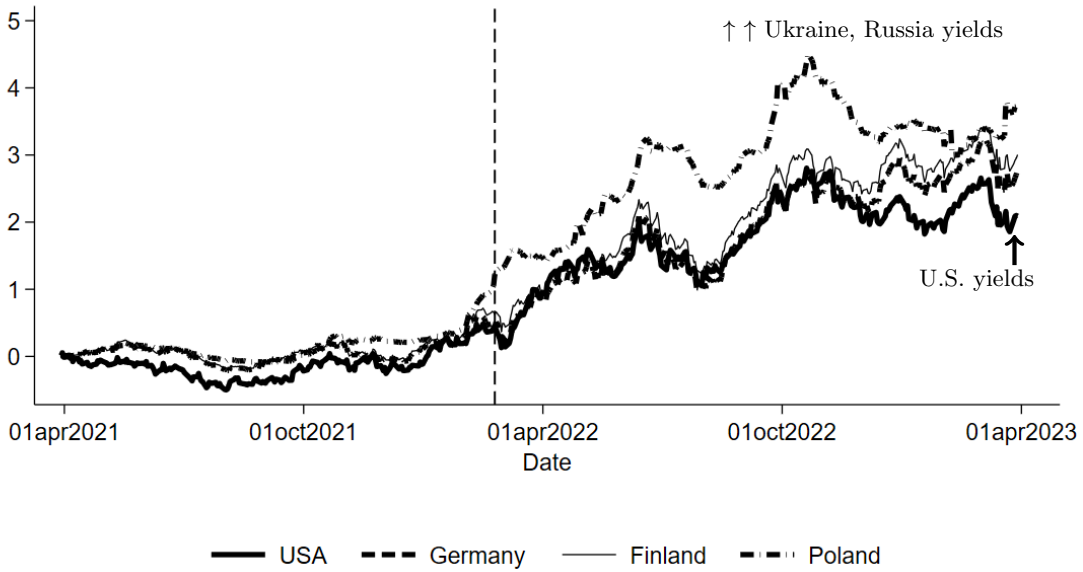
discussions about the withdrawal from Afghanistan caused the newspaper-based tensions index to spike, even though there was no heightened war risk.

Figure 3: Government Bonds around Ukraine Invasion

Panel A: Ukraine and Russia (Bond Prices)



Panel B: U.S. vs. Other Countries (Bond Yields)



Panel A plots scaled daily bond prices from March 30 2021 until March 30 2023 for a Russian and a Ukrainian bond (UKRAIN 7 $\frac{3}{4}$ 09/01/2029, RUSSIA 7 $\frac{1}{2}$ 03/31/2030), both of which are denominated in U.S. Dollars. Both bond prices are normalized to 100 during the five trading days prior to March 30 2021. A vertical line indicates the surprise invasion of Ukraine by Russia on February 24, 2022. Panel B shows daily bond yields for the US, Germany, Finland and Poland, which are inversely related to prices. Bond yields in Panel B are normalized to zero during the five trading days prior to March 30 2021.

Panel B of Figure 3 turns to bond yields for the U.S. and other countries that were not directly affected by the invasion, but nonetheless experienced an increase in geopolitical tensions. It shows that during the days immediately after the invasion, U.S. and other safe developed country bond yields did not increase, and in fact declined. While global yields during the months after the invasion followed an upward trend due to other factors, the relative development of borrowing costs lines up with military capacity and exposure to the war in Ukraine. In particular, at the end of the first quarter of 2023, as the battle for Kyiv came to a conclusion, the U.S. had the lowest yields, followed by (in this order) Germany, Finland, and Poland, with Ukrainian and Russian yields off the charts. Overall, this evidence shows that a shock to geopolitical tensions widens the funding advantage of militarily stronger vs. weaker countries.

2.3 Post-War Devaluations

The final empirical pattern that our model is designed to match is that countries that are defeated in war devalue their debts by more than those that come out relatively unscathed. This empirical fact suggests that bonds are risky because the defeated country is likely to default in states where investors' marginal utility is particularly high, and is linked to the evidence in Figures 2 and 3 if expectations of future devaluations are priced ex ante.

Table 1 shows the average inflation rates for the U.S. and defeated countries for the ten years following World Wars I and II. Inflation in the U.S. was significantly lower than that in the defeated countries. For example, while the U.S. did experience inflation following WWI, it subsequently experienced deflation, resulting with a similar price level after the war as before.¹⁴

This pattern is mirrored by the experience of currency debasement during the earlier 19th century, when the U.K. was the militarily strongest country. In particular, Austria, Russia, and Turkey experienced a cumulative decline in silver content of 50 percent or more during the 19th century, most of it realized during the Napoleonic wars of 1799-1815 (Reinhart and Rogoff (2009)). By contrast, Great Britain's currency held its silver content relatively steady, experiencing a cumulative decline in silver content of only 6 percent over the 19th century.¹⁵ Overall, the experience of inflation and debasement during the periods of U.K. and U.S. hegemony suggest that government bond payoffs were significantly related to who was likely to suffer most in a war.

3 Two-Period Example

To facilitate the discussion of equilibrium uniqueness and multiplicity, we begin by presenting a simple two-period example. Our complete model involves two countries in an infinite horizon

¹⁴We exclude countries with data gaps, such as the Ottoman Empire after World War I and Hungary after World War II from Table 1. This choice is conservative in that these defeated countries generally experienced high inflation.

¹⁵See Reinhart and Rogoff (2009), Table 11.2. The samples for Germany and Portugal in that table are significantly shorter, so we exclude them from our discussion. Even the U.S. experienced one significant episode of default through inflation during the 19th century, with inflation peaking at 24 percent in 1864 after the Civil War.

Table 1: Inflation and Geopolitical Conflict in the 20th Century

| Avg. 10-Year Inflation Rate | post-WWI | post-WWII |
|-----------------------------|----------|-----------|
| U.S. | 1.68 | 4.20 |
| Avg. Defeated Countries | 3.84E+08 | 33.04 |

This table reports 10-year post-war inflation rates in an annualized percent for the U.S. vs. an equal-weighted average of defeated countries. For the 10 years post World War I (1919-1928), the included defeated countries are Austria and Germany. For the 10 years post World War II (1946-1955), the included defeated countries are Austria, Germany, Italy, and Japan. U.S. monthly CPI inflation data is from Robert Shiller’s website. All other CPI inflation data is from Global Financial Data.

framework that make decisions over military and public goods spending, borrowing, and default in every period.

3.1 Environment

There are two time periods $t = 0, 1$. There are two countries $i = 1, 2$, each of mass 1, and risk-neutral global investors with deep pockets. For simplicity, we assume the two countries are identical in every dimension other than exogenous military technology.¹⁶

The date 0 government budget constraint of country i is

$$g_{i0} + m_i = \tau + q_i b_i,$$

where $g_{i0} \geq 0$ is non-military government spending, m_i is military spending, $\tau > 0$ is an exogenous tax revenue, q_i is the global bond price for country i bonds, and $b_i \geq 0$ is the level of borrowing. The date 1 government budget constraint of country i under peace is

$$g_{i1} = \tau (1 - \chi d_i) - b_i (1 - d_i),$$

where $g_{i1} \geq 0$, $d_i = \{0, 1\}$ is country i ’s default decision, and $\chi \in (0, 1]$ is a default cost proportional to the tax revenue of the government. The parameter χ can be interpreted as a reduced form representation of a country’s debt capacity resulting from the de jure and de facto constraints on countries defaulting on their debt. While we do not need to take a stance on the precise drivers of χ , North and Weingast (1989) argued that the Glorious Revolution imposed new constraints on British monarchs and thereby deepened the British bond market. Other potential drivers include the development of a professional tax collection bureaucracy (e.g, Clark (1996), Stasavage (2007)).

The date 1 budget constraint under war if the country wins is identical to that under peace. However, if the country loses the war, the tax endowment is destroyed and the budget constraint

¹⁶While we discuss our model in the context of two countries that are adversarial to one another (e.g., U.S. and China in the present), an equivalent interpretation of our model involves two countries that are allied with one another (e.g., U.S. and Great Britain in the world wars), where the crucial force in the model is that the militarily more dominant country is more likely to repay its debt once the war is over.

becomes

$$g_{i1} = -b_i(1 - d_i).$$

Note that this implies that if country i loses the war, then it defaults on its debt since default is costless. This feature is consistent with the third empirical observation that losers from wars devalue their debt by more than winners.¹⁷ The probability of war $\phi \in (0, 1)$ is assumed to be exogenous and constant.¹⁸

Country i 's probability of winning the war is $w_i(m_i, m_{-i})$ which is increasing in m_i and decreasing in m_{-i} . Throughout our paper, we consider the difference contest function of [Hirshleifer \(1989\)](#):

$$w_1(m_1, m_2) = \frac{\exp(A + m_1)}{\exp(A + m_1) + \exp(m_2)} \equiv F(A + m_1 - m_2). \quad (1)$$

This implies that the probability of winning for country 2 equals

$w_2(m_2, m_1) = 1 - F(A + m_1 - m_2)$. The parameter $A \geq 0$ denotes the exogenous military advantage of country 1 due to factors such as geography or technology. The case with $A = 0$ corresponds to the situation where both countries have equal chances of dominating in a military conflict under symmetric military spending.

The preferences of country i are

$$\mathbb{E}\{g_{i0} + \kappa g_{i1} - \lambda\}.$$

The discount factor κ takes a value of 1 under peace and a value of $\theta > 1$ under war, capturing the fact that the marginal utility of resources under war is higher.¹⁹ We refer to the parameter θ as the war premium. The parameter λ equals 0 under peace or war with victory and $\gamma > 0$ under war with defeat, representing the relative cost of losing a war.

Deep-pocketed international investors have preferences

$$\mathbb{E}\{c_0 + \kappa c_1\},$$

where c_t represents the consumption of investors, which is exogenous.

The order of events is as follows. At date 0, both countries simultaneously choose g_{i0} and b_i , financial markets open and clear, and countries choose m_i to satisfy the government budget

¹⁷In the two-period model, default is necessary for feasibility of non-negative government spending. In the infinite horizon model of the next section, default is potentially not necessary for feasibility, but it is strictly optimal.

¹⁸We choose this benchmark not only because of its analytical simplicity, but also because the empirical evidence is unclear on whether armament impacts the likelihood of war ([Diehl and Kingston \(1987\)](#), [Wallace \(1979\)](#)). However, we have considered extensions in which the likelihood of war increases when military armament is more unequal, capturing the link between the decision of going to war and the expected cost of doing so formalized by [Thoenig \(2024\)](#). Endogenizing the war probability in this way further broadens the scope for equilibrium multiplicity, but otherwise leaves the central mechanism in this paper unchanged.

¹⁹The two-period model results are robust in an environment with concave preferences, but this nonlinearity would make the fully dynamic model non-tractable.

constraint. This timing convention ensures that there is a close link between borrowing capacity and military spending.²⁰

3.2 Equilibrium

We solve the program by backward induction. Under war with defeat at date 1, country i defaults on any outstanding debt. Under war with victory or under peace, it repays its debt only if $b_i \leq \tau\chi$. Thus, $\tau\chi$ captures the maximal amount of debt a country can borrow. For any $b_i \in (0, \tau\chi]$, bond prices satisfy

$$q_i(m_i, m_{-i}) = 1 - \phi + \phi\theta w_i(m_i, m_{-i}). \quad (2)$$

Observe that the bond price depends on the probability of war ϕ , the war premium θ , and the likelihood of victory $w_i(m_i, m_{-i})$.²¹ The difference in bond prices and borrowing costs in our model reflects investors' willingness to pay for insurance, similar to [Gourinchas and Rey \(2022\)](#). However, in a sample with no realized wars, the safety premium in our model would be observationally equivalent to a "convenience yield". "Convenience yields" due to trading frictions and collateral constraints favoring the safer asset would further enhance differences in borrowing rates.

Substituting the bond pricing equation into the country's budget constraint and into its preferences, country i 's optimization program, assuming that it chooses $b_i > 0$, can be written as

$$\max_{m_i, b_i \in (0, \tau\chi]} \left\{ \begin{array}{l} \tau + (1 - \phi + \phi\theta w_i(m_i, m_{-i})) b_i - m_i \\ + (1 - \phi + \phi\theta w_i(m_i, m_{-i})) (\tau - b_i) \\ - \phi\theta (1 - w_i(m_i, m_{-i})) \gamma \end{array} \right\} \quad (3)$$

subject to

$$\tau + (1 - \phi + \phi\theta w_i(m_i, m_{-i})) b_i \geq m_i, \quad (4)$$

where the last constraint corresponds to the non-negativity constraint on non-military government spending at date 0. Observe that the terms in the objective function involving debt cancel out. This is because the government and international lenders have the same preferences, and the revenue raised from bond issuance at date 0 equals the expected payment to international lenders at date 1. Thus, if the non-negativity constraint on non-military spending does not bind for a country, the equilibrium choice of debt is irrelevant and independent of military spending, which is a reflection of Ricardian Equivalence ([Barro \(1974\)](#)). To ensure an interaction between military spending and debt and thereby break Ricardian Equivalence, we assume that the cost of war γ is sufficiently large that debt capacity constrains military spending.²² This is consistent with the empirical observation that military buildups are typically debt-financed ([Federle, Marzian and Trebesch \(2024\)](#)) and the

²⁰The staggered timing of the choices of g_{i0} (which must be non-negative) and m_i is necessary to ensure that the government budget constraint is satisfied off the equilibrium path, with off-equilibrium changes in bond prices being absorbed by changes in military spending.

²¹Our model thus delivers a positive average risk premium on risky government bonds, which is consistent with long-run evidence on bond risk premia documented by [Meyer, Reinhart and Trebesch \(2022\)](#).

²²There is a finite threshold for γ above which our results hold; we take this value to infinity to reduce notation.

majority of discretionary spending in the U.S. (which is debt-financed on the margin) is military.

Assumption 1. $\gamma \rightarrow \infty$.

Under this assumption, the cost of losing a war is sufficiently severe that both countries choose zero spending on public goods at date 0.²³ Since constraint (4) binds for both countries, it follows that $b_1 = b_2 = \tau\chi$. As such, one can interpret the parameter χ as capturing the total amount of liquidity demand in international financial markets. Since (4) holds with equality for both countries, it follows that these can be combined, taking into account the functional form for the contest function (1) and bond pricing equation (2):

$$\underbrace{m_1 - m_2}_{\text{Gap in military expenditure}} = \tau\chi \times (q_1 - q_2) = \underbrace{\tau\chi \times \phi\theta (2F(A + m_1 - m_2) - 1)}_{\text{Gap in bond market revenue}}. \quad (5)$$

Equation (5) states that in equilibrium, the gap in military expenditure (i.e. military hegemony) must match the difference in bond market revenue (“exorbitant privilege” of the hegemon).

3.3 Uniqueness and Multiplicity

We now examine what condition (5) implies for the uniqueness or multiplicity of equilibria. We use m_i^* to denote the equilibrium values for country i ’s military investment and q_i^* for the equilibrium bond price. In this two-period model, we define equilibrium stability following [Fudenberg and Tirole \(1991, p. 24\)](#), where a Nash equilibrium is considered stable if an iterative tatonnement adjustment process in its neighborhood leads back to the same equilibrium. An analogous definition holds here if we consider a tatonnement process where the bond market moves in alternation with the debt-issuing countries.

Proposition 1 *There exists a debt capacity threshold $\chi' > 0$ that is a positive function of the exogenous advantage A and a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

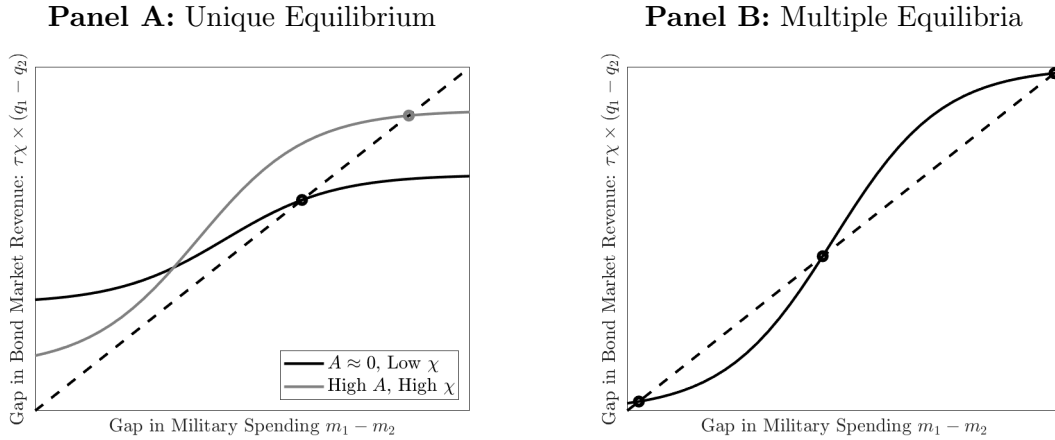
- i) *If $\chi < \chi'$, then the equilibrium is unique. If country 1 does not have an exogenous military advantage, i.e. $A = 0$, then the equilibrium is symmetric with $w_1(m_1^*, m_2^*) = \frac{1}{2}$. If country 1 has an exogenous military advantage, i.e. $A > 0$, then the equilibrium is asymmetric with country 1 dominating militarily and financially: $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$, and $q_1^* > q_2^*$.*
- ii) *If $\chi > \chi'$, then there are two stable equilibria and one unstable equilibrium. In one stable equilibrium, country 1 dominates militarily and financially: $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$, and $q_1^* > q_2^*$. In the other, country 2 dominates militarily and financially: $w_1(m_1^*, m_2^*) < \frac{1}{2}$, $m_1^* < m_2^*$, and $q_1^* < q_2^*$.*

Figure 4 Panel A illustrates two examples in which the equilibrium is unique and the condition of Proposition 1 is satisfied. It depicts equation (5), which shows that the difference in military

²³Note that one can interpret the level of equilibrium government spending, which is zero in our model, to reflect necessary or pre-committed public goods.

spending between countries 1 and 2 (shown on the x-axis) must match the difference in their revenue from international bond markets (shown on the y-axis). This revenue difference is influenced by the relative probability of winning the war, which in turn is a function of how much each country spends on its military. The dotted line corresponds to the 45-degree line and the black and gray solid lines correspond to the gap in bond market revenue as a function of the gap in military capacity in two different cases. In each case, an equilibrium corresponds to the intersection of the solid line and the 45-degree line.

Figure 4: Equilibria in Two-Period Model



This figure plots the equilibrium condition (5) for different cases. A solid line depicts the gap in bond market revenue. An equilibrium is defined by the intersection of the solid line and the dotted 45-degree line. Panel A depicts two examples of unique equilibria and Panel B depicts an example of multiple equilibria. In Panel A, the solid black line represents the gap in bond market revenue under a low exogenous military advantage (i.e. $A \approx 0$). Debt capacity is sufficiently low that the slope of the curve is always less than one. The solid gray line represents the gap in bond market revenue under a high exogenous military advantage $A \gg 0$ and high debt capacity. The maximum slope of this curve exceeds one, but the exogenous military advantage A is large enough to ensure equilibrium uniqueness. Panel B depicts the gap in bond market revenue under a small exogenous military advantage (i.e. $A \approx 0$) and high debt capacity $\chi > \chi'$, so that the maximum slope of this curve exceeds one and there are multiple equilibria.

The solid black line corresponds to the case for which the military advantage for country 1 is small with $A \approx 0$. Debt capacity is sufficiently small that the slope of the revenue gap curve is below one for all values of $m_1 - m_2$. In this case, the bond market revenue gap increases only weakly with the gap in military expenditures. As a result, there is only one point of intersection between the solid and dotted lines, and the equilibrium is unique. Country 1 invests relatively more in the military, wins the war in expectation, faces lower funding costs, and raises more revenue in the bond market.

The solid gray line corresponds to the case for which there is a sizable military advantage for country 1 so that $A \gg 0$, ensuring uniqueness. In this case, debt capacity is sufficiently high that the slope of the bond market revenue gap curve exceeds one in some range of $m_1 - m_2$. Because debt capacity is sufficiently large, relative bond market revenue depends more than one-for-one on relative military spending for some range of relative military spending. However, despite this, there

is only one point of intersection between the solid and dotted lines because the military advantage (which shifts the solid line) is sufficiently large.

Figure 4 Panel B illustrates an example of multiplicity in the case where the military advantage for country 1 is small with $A \approx 0$. Recall that the value of χ' is an increasing function of A , meaning that a small value for A facilitates equilibrium multiplicity. Because $\chi > \chi'$, the slope of the revenue gap curve exceeds one for some values of $m_1 - m_2$, similar to the second case in Panel A. In contrast to that case, the exogenous advantage for country 1 in Panel B is not large, so the bond market revenue curve is lower than the gray line in Panel A, leading to the emergence of multiple equilibria. Intuitively, even though there exists an equilibrium where the exogenously stronger country wins the war in expectation and benefits from a funding advantage, there is another equilibrium where the exogenously weaker country can overwhelm the stronger country with a sufficiently large military investment. Bond markets rationally anticipate a higher probability of victory for the weaker country in this equilibrium, thus providing it with a funding advantage that reinforces its military advantage.²⁴ Observe that two asymmetric stable equilibria can be obtained even if neither country has an exogenous military advantage ($A = 0$). In that case, there are two stable equilibria where either country 1 or country 2 dominates, despite the fact that neither country has an exogenous military advantage.

The examples of uniqueness and multiplicity clarify that our model is consistent with the first and second empirical facts discussed in the previous section. The country that wins the war in expectation in our model also benefits from a funding advantage since it faces lower borrowing costs. Moreover, this funding advantage rises in the probability of war, holding relative military spending fixed. This is clear from the bond market pricing equation (2) that shows that the difference between bond prices is rising in the war probability ϕ when military spending is held constant. The following corollary expands on these ideas by examining the complementarity between military success and funding advantage.

Corollary 1 *Consider the equilibrium comparative statics in the neighborhood of the stable equilibrium in which country i dominates. Country i 's probability of winning the war $w_i(m_i^*, m_{-i}^*)$ and funding advantage $q_i^* - q_{-i}^*$ both increase in debt capacity χ , in the war probability ϕ , the war premium θ , and in country i 's exogenous military advantage.*

The expected winner's military and financial dominance are amplified by several features. For example, as debt capacity increases, so does the victor's ability to borrow and invest in military capacity. This ability, in turn increases the probability of winning the war, which reduces that country's likelihood of default. Thus, at higher debt capacity, international investors are even more willing to hold the expected winner's bonds, which increases their price and reduces the relative funding costs of the expected winner.

²⁴Our multiplicity result does not rely on the choice of a difference contest function which is convex-concave. We have verified that multiplicity also arises with ratio contest functions which are concave, or with linear contest functions and constraints on military capacity. See Online Appendix C. See [Hirshleifer \(1989\)](#) for a discussion of contest functions.

An analogous logic holds if the probability of war or the war premium increase. In both cases, the dominant country's funding advantage rises holding relative military spending fixed. In response, the dominant country invests relatively more in the military while the defeated country invests relatively less, thus amplifying the dominant country's military and financial dominance.

A related observation involves the feedback loop from military advantage to funding advantage. Consider an increase in the exogenous advantage A starting from an equilibrium in which the exogenously stronger country is expected to win a war. Holding relative military spending fixed, an increase in the exogenous military advantage increases the funding advantage of the stronger country since it increases its likelihood of victory and reduces its likelihood of default. In response, the stronger country invests relatively more in the military, which further boosts its geopolitical and funding advantage.

We conclude our analysis of the two-period model with one final observation relating to the fact that the cutoff χ' is a negative function of the war probability ϕ and the war premium θ , as stated in Proposition 1. Combined with Corollary 1, this means that financial market factors that enhance the complementarity between military and financial dominance for the exogenously stronger country—such as higher debt capacity, war probability, and war premium—also invite the possibility of equilibrium multiplicity, with the weaker country using financial markets to overwhelm the stronger country. In this sense, military strength derived from the ability to borrow in financial markets is inherently different from exogenous military advantage, which, while also enhancing the complementarity between military and financial dominance, reduces the potential for equilibrium multiplicity.

These observations shed light on the role that global debt markets play in generating equilibrium multiplicity. In the absence of any debt markets, countries would be unable to borrow, they would invest their entire endowment τ in the military, and the equilibrium would be unique. A similar pattern would emerge with only domestic borrowing from non-deep-pocketed investors; in that case, both countries would borrow up to their domestic borrowing capacity and the equilibrium would be unique.²⁵ In contrast, in a global debt market, both countries compete jointly for resources from deep-pocketed investors, and the possibility of multiplicity arises.

4 Infinite Horizon Analysis

We now examine our results in an infinite horizon environment in which peace or war can occur in every date. We begin by showing that the insights regarding equilibrium multiplicity in the two-period model translate to a dynamic environment, in which equilibrium multiplicity now corresponds to the multiplicity of steady states conditional on the two countries remaining at peace. Moreover, we evaluate transition dynamics around those steady states and examine the conditions

²⁵More specifically, suppose that households have an endowment e that they can lend to the government. It follows that in equilibrium, $q_{it}b_{it} = e$, with both countries borrowing maximally, and military spending equal to $\tau + e$. As long as some borrowing can be done globally, the equilibrium is as in the case with deep-pocketed international investors.

under which those dynamics are uniquely determined. We use this characterization to delineate conditions under which a hegemonic transition from one peaceful steady state to another is possible.

4.1 Environment

There are periods $t = 0, 1, \dots$ and three potential states of the world: peace, war with victory for country 1, or war with defeat for country 1. The date t government budget constraint of country i under peace is

$$g_{it} + m_{it} = \tau (1 - \chi d_{it}) + (1 - \delta) m_{it-1} + q_{it} b_{it} - b_{it-1} (1 - d_{it}).$$

As in the two-period model, $g_{it} \geq 0$ is non-military government spending, $\tau > 0$ is an exogenous tax revenue, q_{it} is the global bond price for country i bonds, $b_{it} \geq 0$ is the face value of one-period debt issued at time t , $d_{it} = \{0, 1\}$ is country i 's default decision at time t , and $\chi \in (0, 1]$ is a default cost as a share of tax revenue. Note that in contrast to the two-period model, m_{it} , which we refer to as military capital, is accumulated over time, with a depreciation rate of $\delta \in (0, 1)$, so that $m_{it} - (1 - \delta) m_{it-1}$ is period t military spending.

The government budget constraint under war with victory is

$$g_{it} + m_{it} = \tau (1 - \chi d_{it}) + q_{it} b_{it} - b_{it-1} (1 - d_{it}),$$

where we have assumed that military capital is destroyed during war. The budget constraint under war with defeat is

$$g_{it} + m_{it} = q_{it} b_{it} - b_{it-1} (1 - d_{it}),$$

where again military capital is destroyed in war. As in the two-period model, if the country loses the war, we assume that its tax endowment is also destroyed.

The probability of war in every date is exogenous and equal to $\phi \in (0, 1)$. Country i 's probability of winning the war at $t + 1$ is $w_i(m_{it}, m_{-it})$, with $w_1(m_{1t}, m_{2t}) = F(A + m_{1t} - m_{2t})$ and $w_2(m_{2t}, m_{1t}) = 1 - F(A + m_{1t} - m_{2t})$ as defined in the contest function in (1).

There are overlapping generations of governments that make decisions at date t , where the date t government has preferences represented by:²⁶

$$\mathbb{E} \{g_{it} + \kappa_{t+1} g_{it+1} - \lambda_{t+1}\}.$$

κ_{t+1} takes a value of 1 under peace at $t + 1$ and a value of $\theta > 1$ under war, with θ corresponding to the war premium as in the two-period model. λ_{t+1} takes a value of 0 under peace or war with victory at $t + 1$ or a value of $\gamma > 0$ under war with defeat. This parameter captures the cost of

²⁶If instead governments were infinitely lived, additional strategic considerations involving coordination across countries would need to be taken into account, leading to even more equilibrium multiplicity than what we highlight in this simpler case. Because our countries are assumed to issue only short-term debt, we also rule out equilibria that depend on expectations several periods ahead.

losing a war.

International investors at date t have preferences

$$\mathbb{E} \{c_t + \kappa_{t+1} c_{t+1}\},$$

where c_t represents the consumption of investors.

At any date t , both countries simultaneously choose $\{g_{it}, b_{it}, d_{it}\}$, financial markets open and clear, and countries choose m_{it} to satisfy the government budget constraint.

4.2 Equilibrium

Because each government solves a two-period problem, we can solve the program by backward induction. Under war with defeat at date t , country i defaults on any outstanding debt by the same logic as in the two-period model. Moreover, under war with victory or under peace, it repays its debt only if $b_{it} \leq \tau\chi$, and therefore $\tau\chi$ captures the maximal amount of debt a country can borrow. For any $b_{it} \in (0, \tau\chi]$, bond prices satisfy the analog of (2):

$$q_{it}(m_{it}, m_{-it}) = 1 - \phi + \phi\theta w_i(m_{it}, m_{-it}). \quad (6)$$

Now consider the optimization problem of a government at date t . By the same logic as in the two-period model, under Assumption 1 that states that $\gamma \rightarrow \infty$, it follows that $g_{it} = 0$ and $b_{it} = \tau\chi$ for all t . The dynamic budget constraint under peace at t can thus be rewritten as

$$m_{it} = \tau + (1 - \delta) m_{it-1} - (\phi - \phi\theta w_i(m_{it}, m_{-it})) \tau\chi. \quad (7)$$

The budget constraints for countries 1 and 2 can be combined, taking into account the functional form for the contest function, and defining the gap in military capital $\mu_t \equiv m_{1t} - m_{2t}$ to yield:

$$\underbrace{\mu_t - (1 - \delta) \mu_{t-1}}_{\text{Gap in military expenditure}} = \underbrace{\tau\chi \times \phi\theta (2F(A + \mu_t) - 1)}_{\text{Gap in bond market revenue}}. \quad (8)$$

Equation (8) is analogous to equation (5) since it states that in equilibrium, the gap in military expenditure between countries is proportional to the gap in bond market revenue. Relative to the two-period model, that gap in military expenditure is a function of the gap in military capital at t as well as a $t - 1$.

We can write the equivalent of condition (8) under war with victory for country 1 as

$$\mu_t = \tau(1 - \chi) + \tau\chi \times \phi\theta (2F(A + \mu_t) - 1), \quad (9)$$

with an analogous condition that would hold under war with defeat for country 1:

$$\mu_t = -\tau(1 - \chi) + \tau\chi \times \phi\theta (2F(A + \mu_t) - 1). \quad (10)$$

During peace at t , condition (8) defines a correspondence from μ_{t-1} to μ_t which may or may not be one-to-one. During war at t with victory for country 1, condition (9) defines a value for μ_t that is independent of history. Condition (10) defines an analogous value for μ_t during war at t with defeat for country 1. Together, these three equations characterize equilibrium dynamics for μ_t .

4.3 Uniqueness and Multiplicity

We define a steady state μ_{ss} as the level of μ_t to which the equilibrium converges after an infinite sequence of peaceful states. We focus our attention on monotonically stable steady states, that is, a steady state value of μ_{ss} for which the transition dynamics under peace defined by equation (8) admit a monotonic transition path for μ_t in the neighborhood of μ_{ss} .

Proposition 2 *There exists a debt capacity threshold χ' that is a positive function of the exogenous advantage A and the depreciation rate δ and a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

- i) *If $\chi < \chi'$, then the steady state is unique and is monotonically stable. If country 1 does not have an exogenous military advantage, i.e. $A = 0$, then the steady state is symmetric with $w_1(m_{1ss}^*, m_{2ss}^*) = \frac{1}{2}$. If country 1 has an exogenous military advantage, i.e. $A > 0$, then the steady state is asymmetric with country 1 dominating militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) > \frac{1}{2}$, $m_{1ss}^* > m_{2ss}^*$, and $q_{1ss}^* > q_{2ss}^*$.*
- ii) *If $\chi > \chi'$, then there are two monotonically stable steady states and one monotonically unstable steady state. In one monotonically stable steady state, country 1 dominates militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) > \frac{1}{2}$, $m_{1ss}^* > m_{2ss}^*$, and $q_{1ss}^* > q_{2ss}^*$. In the other, country 2 dominates militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) < \frac{1}{2}$, $m_{1ss}^* < m_{2ss}^*$, and $q_{1ss}^* < q_{2ss}^*$.*

Figure 5 Panel A illustrates an example with a unique steady state. The x-axis depicts the value of μ_{t-1} and the y-axis depicts the value of μ_t . The solid line shows the correspondence defined by condition (8), and the dotted line is the 45-degree line. The intersection of these two lines is a steady state, and in this example, there is a single intersection and a unique steady state. The logic for this result is similar to the two-period case. Because debt capacity is below χ' , the exogenously weaker country is unable to use financial markets to overwhelm the stronger country, and the exogenously stronger country enjoys geopolitical and funding advantages that complement each other.

Figure 5 Panels B and C illustrate two examples with multiple steady states. In both examples, the correspondence defined by condition (8) intersects the 45-degree line three times, with each intersection depicting a steady state. The middle steady state is not monotonically stable, but the steady states to the right and to the left of the middle steady state are monotonically stable.²⁷ Note

²⁷In both panels, the transition path diverges away from the middle steady state. There are also examples where the slope of the correspondence at the middle steady state is between 0 and -1, and convergent transition paths around the middle steady state exist; however, such paths feature oscillations and are thus not monotonic.

that in the example in Panel B, the correspondence from μ_{t-1} to μ_t is one-to-one, whereas this is not the case in Panel C. We will return to this difference in the next subsection in our discussion of transition dynamics.

In each of the examples depicted in Panels B and C, debt capacity is sufficiently large that the exogenously weaker country is able to overwhelm the stronger country by borrowing in financial markets. As in the two-period model, this multiplicity is facilitated by a higher probability of war, a higher war premium, and a smaller exogenous advantage for country 1. In addition, Proposition 2 shows that a lower depreciation rate for military capital increases the scope for multiple steady states by making it more feasible for the exogenously weaker country to accumulate military capital over time, and thereby overwhelm the stronger country in steady state.

While not formally repeated here, the comparative statics for equilibria in the two-period model in Corollary 1 also translate directly to steady states of the dynamic model. Hence, in a steady state where country i dominates, the steady-state funding advantage increases in the debt capacity χ , the war probability ϕ , the war premium θ , and country i 's exogenous advantage. As before, a stronger ability to tap international bond markets or a stronger exogenous military advantage amplify the dominant country's funding and military advantage.

4.4 Transition Dynamics

Each panel in Figure 5 depicts a monotonic transition path in the neighborhood of the monotonically stable steady state where the exogenously stronger country dominates. The slope of the mapping from μ_{t-1} to μ_t at the intersection defining such a steady state is between zero and one; thus, a convergent transition path around the steady state exists. It is clear that in the case of a unique steady state in Panel A, such a transition path is globally unique: starting from any value of μ_0 , multiple consecutive periods of peace cause μ_t to converge towards the steady-state value μ_{ss} . For instance, if $\mu_0 < \mu_{ss}$, then μ_t rises over time along with the military and financial advantage of country 1.

We now turn to transition dynamics in the more complicated cases with multiple steady states.

Proposition 3 *Suppose that $\chi > \chi'$ so that there are two monotonically stable steady states. Then there exists a debt capacity threshold χ'' that is a negative function of the war probability ϕ and the war premium θ that satisfies the following properties:*

- i) (**geopolitical hysteresis**) If $\chi < \chi''$, then there does not exist any non-monotonic peaceful transition path to a monotonically stable steady state, and initial conditions determine the steady state.*
- ii) (**geopolitical fragility**) If $\chi > \chi''$, then there exists a non-monotonic peaceful transition path to each of the monotonically stable steady states, and the same initial conditions can lead to different steady states.*

Figure 5 Panel B depicts an example of *geopolitical hysteresis* with $\chi' < \chi \leq \chi''$. Since the correspondence from μ_{t-1} to μ_t is one-to-one, the transition path under consecutive peaceful periods is uniquely determined starting from any initial condition μ_0 . If μ_0 is sufficiently high, then the equilibrium converges to the steady state in which country 1 dominates militarily and financially. The opposite occurs if μ_0 is sufficiently low. The relative levels of military and financial dominance at a point in time therefore determine the identity of the eventually dominant country after many periods of peace, as well as the convergence path.

Figure 5 Panel C depicts a situation of *geopolitical fragility* with $\chi > \chi'' > \chi'$. There still exists a monotonic path towards each of the monotonically stable steady states. However, there also exist non-monotonic paths that begin within the zone of geopolitical fragility, defined as the range where the correspondence from μ_{t-1} to μ_t is one-to-many. The zone of fragility is where the starting gap in military capacity μ_{t-1} is neither too large nor too small. Within this zone, there can be multiple peaceful transition paths leading to either steady state. In this example, the relative military and financial dominance at a point in time need not determine the eventual identity of the dominant country after multiple periods of peace.

Within the zone of fragility, the different possible equilibrium paths arise because of the interaction between financial markets and military capacity. Forward-looking rational investors link the “exorbitant privilege” of the military hegemon to its military dominance. Market expectations can change quickly, causing the erosion of a country’s previous financial and military dominance. Once the equilibrium transitions out of the zone of fragility, it settles on a convergence path towards a new steady state, with the new expected winner establishing greater and greater military and financial dominance over time.

Higher debt capacity matters for this analysis because it makes geopolitical fragility more likely than hysteresis. The intuition is that when debt capacity is high, bond market coordination around a potential victor has a much greater effect on countries’ ability to raise financing, amplifying the effect on relative military strength. Thus, the factors that make multiple steady states more likely—high debt capacity, high war probability, and high war premium—also make geopolitical fragility more likely.

Using this proposition, we now investigate conditions under which a hegemonic transition can occur. We define a hegemonic transition as a transition path—which may include the realization of war—that begins in one monotonically stable steady state but eventually converges to the other monotonically stable steady state.

Corollary 2 *Suppose that $\chi > \chi'$ so that there are two monotonically stable steady states.*

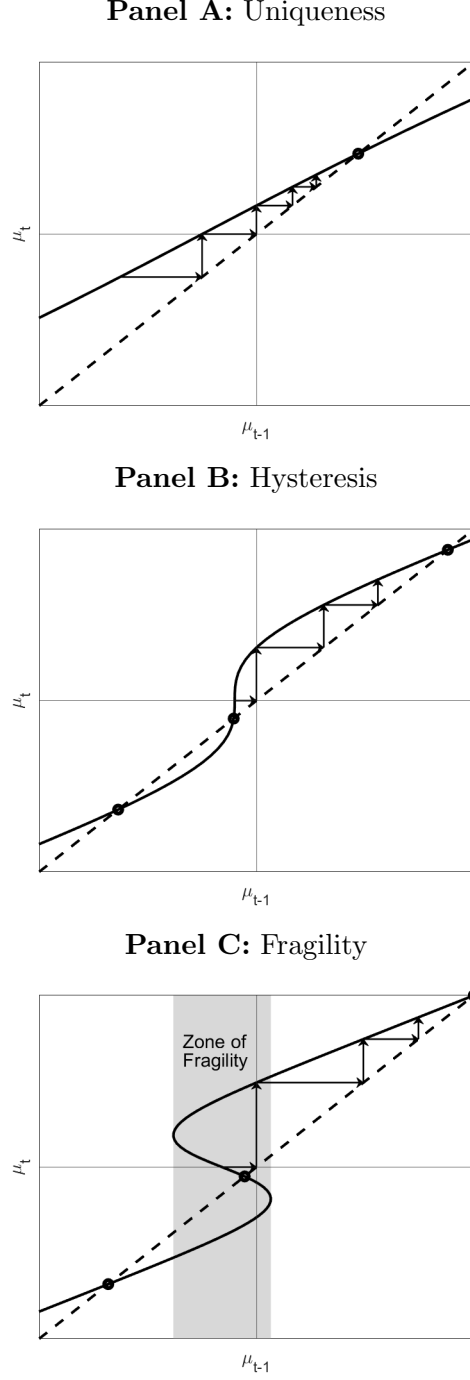
- i) If $\chi < \chi''$, then a hegemonic transition requires war.*
- ii) If $\chi > \chi''$, then there exists a hegemonic transition path that avoids war if the depreciation rate δ is sufficiently high.*

The logic for the first part of the corollary stems from the first part of Proposition 3. Any path that avoids war is predetermined by initial conditions μ_0 , thus implying that a hegemonic transition

in the absence of war is not possible. To see how war could lead to a hegemonic transition in this case, consider the example in Figure 6, Panel A where we have depicted a situation in which the military advantage of country 1 is low with $A \approx 0$. Suppose that at date 0, the equilibrium begins in the steady state in which country 1 dominates. Suppose that war takes place at date 1, and suppose that country 1 loses the war. At date 1, military capital is destroyed for both countries, and the tax revenue of country 1 is also destroyed, with μ_1 now satisfying condition (10). Given that $A \approx 0$, it follows that $\mu_1 < 0$, so that country 2 now dominates militarily and financially. Moreover, it follows that any peaceful transition path necessarily converges to the steady state in which country 2 is the hegemon. This example illustrates a situation in which a hegemonic transition is made possible by the realization of war and defeat of the exogenously stronger country, similar to the historical transitions depicted in Figure 1.

The second part of the corollary shows that if debt capacity is sufficiently high, a second type of hegemonic transition is possible, originating from financial markets and without the realization of war. It directly follows from the second part of Proposition 3. To see how a hegemonic transition can occur in peacetime, consider the example in Figure 6, Panel B under a small exogenous advantage $A \approx 0$. Observe that in this example, the zone of fragility is sufficiently wide that it encompasses both steady states. Suppose that at date 0, the equilibrium begins in the steady state in which country 1 dominates. While there exists a path under peace where both countries remain in this steady state, there also exists a path under peace that converges monotonically to the steady state in which country 2 is the hegemon, as depicted in Figure 6. Thus, in contrast to the previous case, a hegemonic transition need not require war. This is because country 2's debt capacity is sufficiently large that country 2 can fund itself in international markets to achieve eventual dominance along some transition path. Of course, the second steady state in this example is also in the zone of fragility, meaning that the dynamics can switch between steady states more than once, and peaceful hegemonic transitions can occur repeatedly.

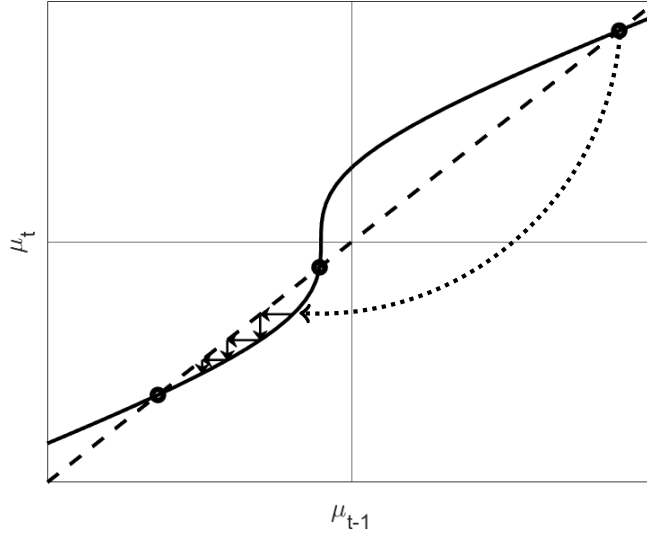
Figure 5: Steady States and Convergence in Dynamic Model



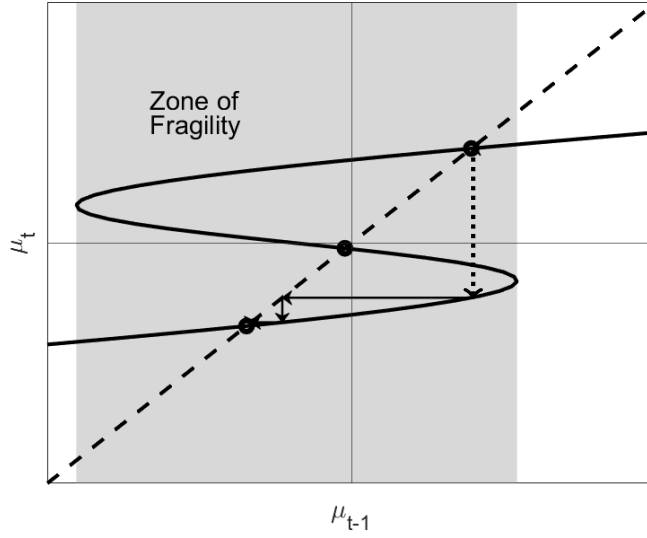
This figure plots the correspondence from μ_{t-1} to μ_t under peace in (8) for three different cases. A steady state is defined by the intersection of the curve and the dotted 45-degree line. Panel A depicts a case for which $\chi \leq \chi'$ and there is a unique monotonically stable steady state. Panels B and C depict cases for which $\chi > \chi'$ and there are two monotonically stable steady states. Panel B depicts the case of geopolitical hysteresis with $\chi < \chi''$. Panel C depicts the case of geopolitical fragility with $\chi > \chi''$. The arrows in each figure depict a monotonic convergent transition path towards a steady state. The “zone of fragility” is defined as the range where the mapping from μ_{t-1} to μ_t is one-to-many.

Figure 6: Hegemonic Transitions

Panel A: War with Defeat



Panel B: Peaceful Transition



This figure plots the correspondence from μ_{t-1} to μ_t under peace in (8) for two different cases. A steady state is defined by the intersection of the curve and the dotted 45-degree line. Panel A depicts a case for which $\chi'' > \chi > \chi'$ and there are two monotonically stable steady states with geopolitical hysteresis. The arrows depict a hegemonic transition during war with defeat for country 1 starting from the steady state in which country 1 dominates. Panel B depicts a case for which $\chi > \chi'' > \chi'$ and there are two monotonically stable steady states with geopolitical fragility. The arrows depict a hegemonic transition under peace starting from the steady state in which country 1 dominates.

Observe that this type of transition occurs if the depreciation of military capital δ is sufficiently high to ensure that the steady states are within the zone of fragility, and debt capacity χ is sufficiently high to ensure that multiple steady states exist. If depreciation is sufficiently high, both countries inherit more similar military capacity from the previous period, facilitating a bond-market coordinated re-set of the military balance. As a result, the steady state is fragile in such an environment. This finding suggests that if the depreciation rate of military technology rises, e.g. due to shifts away from expensive fighter planes towards cheaper replaceable drones, then geopolitical fragility and peaceful hegemonic transitions will be more likely.

4.5 Extension: Asymmetric Debt Capacity

We now consider an extension of our model to a situation in which the two countries have asymmetric debt capacity. This extension is motivated by the observation that Great Britain prevailed during the Napoleonic Wars in large part because of its higher debt capacity. It is also motivated by the question of what China's efforts to develop its international bond markets, such as studied in [Clayton et al. \(2022\)](#), would imply for its geopolitical rivalry with the U.S. To facilitate exposition, we let $A = 0$ so that the only source of asymmetry between the two countries is their relative debt capacity, and we let χ_i correspond to country i 's debt capacity. We let $\chi_1 \geq \chi_2$ so that country 1 has weakly larger debt capacity.

Country i 's budget constraint taking into account that $g_{it} = 0$ and $b_{it} = \tau\chi_i$ can be represented analogously to equation (7) as

$$m_{it} = \tau + (1 - \delta)m_{it-1} - (\phi - \phi\theta w_i(m_{it}, m_{-it}))\tau\chi_i. \quad (11)$$

Combining this equation for countries 1 and 2 yields

$$\mu_t - (1 - \delta)\mu_{t-1} = \left(-\phi + \frac{\phi\theta}{2}\right)\tau(\chi_1 - \chi_2) + \tau\frac{\chi_1 + \chi_2}{2} \times \phi\theta(2F(\mu_t) - 1). \quad (12)$$

This equation shows that relatively higher debt capacity $\chi_1 - \chi_2$ shifts country 1's relative military investment up as long as the war premium θ is sufficiently high, acting similarly to an exogenous military advantage in the previous section. The intuition is that if the probabilities of winning are relatively similar, a relatively higher debt capacity increases the bond market revenue for country 1 relative to country 2.²⁸ The gap in the probability of winning $2F(\mu_t) - 1$ affects military investment through the average debt capacity $\frac{\chi_1 + \chi_2}{2}$, because higher debt capacity for either country increases the spillover from the probability of winning to the gap in bond market revenue.

We now evaluate this equilibrium condition and consider the circumstances under which there is a unique steady state or there are multiple steady states. We consider the case where country 1's debt capacity is sufficiently high and weakly exceeds that of country 2, and where the war premium is sufficiently high such that $\theta > 2$. Moreover, for simplicity, we consider comparative statics for

²⁸The condition on the war premium θ ensures that the average bond price exceeds one, so a country with an even probability of winning raises more funding from new bonds issued than the cost of repaying last period's bonds.

the difference in debt capacity $\chi_1 - \chi_2$, holding total debt capacity $\chi_1 + \chi_2$ fixed. This comparative static can be interpreted as one country deepening its financial market relative to its rival, while global bond market depth remains constant.

Proposition 4 *Suppose that $\chi_1 \geq \chi_2$, $\chi_1 > \frac{4}{\tau\phi\theta}$. For any given value of global debt capacity $\chi_1 + \chi_2$ there exists a debt capacity difference threshold η with the following properties:*

- i) If $\chi_1 - \chi_2 > \eta$, then the steady state is unique and is monotonically stable. The steady state is asymmetric with country 1 dominating militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) > \frac{1}{2}$, $m_{1ss}^* > m_{2ss}^*$, and $q_{1ss}^* > q_{2ss}^*$.*
- ii) If $\chi_1 - \chi_2 < \eta$, then there are two monotonically stable steady states and one monotonically unstable steady state. In one monotonically stable steady state, country 1 dominates militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) > \frac{1}{2}$, $m_{1ss}^* > m_{2ss}^*$, and $q_{1ss}^* > q_{2ss}^*$. In the other, country 2 dominates militarily and financially: $w_1(m_{1ss}^*, m_{2ss}^*) < \frac{1}{2}$, $m_{1ss}^* < m_{2ss}^*$, and $q_{1ss}^* < q_{2ss}^*$.*

This proposition states that even if there is no exogenous military advantage for country 1, country 1 will dominate country 2 militarily if its debt capacity is sufficiently larger than country 2's debt capacity. The country with higher debt capacity endogenously emerges with lower borrowing costs, i.e. it enjoys the “exorbitant privilege”, and is the military hegemon expected to win in a military conflict.²⁹ This result emerges from the complementarity between military and financial power that we have highlighted in previous sections: Greater debt capacity supports a larger military buildup, which generates a funding advantage that further supports the military. This ability to dominate country 2 is eroded if either country 2 increases its debt capacity or alternatively if country 1's own debt capacity is eroded. In both of these circumstances, country 2's ability to raise sufficient funds to overwhelm country 1 is enhanced and multiple steady states can emerge.³⁰

We have also examined extensions that consider other asymmetries, in particular, asymmetries in the tax endowment τ and the war premium θ . Using logic analogous to that of Proposition 4, an increase in asymmetry makes the emergence of a unique equilibrium more likely, with the country commanding the higher tax endowment and the higher war premium dominating militarily and financially. This result emerges because the tax endowment and war premium interact with borrowing capacity to make multiplicity more likely.³¹

²⁹A corollary to this result is that, with a sufficiently high debt capacity, country 1 can dominate country 2 even if it suffers an exogenous disadvantage with $A < 0$.

³⁰Whether the case of multiplicity involves hysteresis or fragility depends on the average of χ_1 and χ_2 . If that average is sufficiently small, then the case of multiplicity admits hysteresis. Otherwise, it admits fragility.

³¹In addition, we have considered other extensions in which country 1's debt enters directly into the welfare of investors, as it would if country 1's debt commands a convenience yield. We can show in an extension employing the functional form estimated by Choi, Kirpalani and Perez (2024) that such a convenience yield would effectively shift up the military capacity of the country earning a convenience yield and serve as a microfoundation for the exogenous advantage, A . This can result in country 1 dominating militarily and financially.

5 Quantitative Exploration

We next provide a simple quantification, which provides additional insights on which model parameters have particularly strong or nonlinear effects on multiplicity and fragility thresholds. Parameters are largely drawn from the rare disasters literature (in particular [Barro \(2006\)](#)), which has shown that a small probability of large consumption declines (such as wars) can reconcile several asset pricing puzzles.³² We set the war probability to 1.7% per year, the consumption decline conditional on a disaster to 35%, and CRRA risk aversion over the disaster state to $\psi = 4$. The implied war discount factor then equals $\theta = (1 - b)^{-\psi} = 5.60$, where b is investors' consumption decline in a disaster. The length of one period is set to 6 years to match the maturity of U.S. government debt, and the depreciation rate is set to a conventional value of 15% annualized ([Cooley and Prescott \(1995\)](#)). For the quantitative exercise, we allow for a discount factor, which is set to $\beta = 0.99$ in annualized units. The exogenous military advantage, A , is set to zero throughout this section. Details of the quantification are provided in the Online Appendix.

Figure 7, Panel A, shows that the fragility and multiplicity thresholds decrease strongly and nonlinearly in both the war probability, ϕ , and investors' risk aversion over the war state, ψ . Recall that a lower threshold means that multiplicity or fragility are possible at lower Debt-to-GDP ratios. As investor risk aversion varies along the y-axis, so does the implied war discount factor $\theta = (1 - b)^{-\psi}$. Both thresholds are scaled by the multiplicity threshold at the baseline parameter values, $\bar{\chi}'$. We see that moving from an annualized war probability of 1.7% to 15% cuts the multiplicity threshold roughly by a factor of six. Setting investor risk aversion to $\psi = 8$ similarly lowers the multiplicity threshold by a factor of six compared to the baseline. While the fragility threshold is higher overall, it scales similarly with the war probability and war risk aversion.

One lesson from our model is that even if fragility is short-lived, this may change the initial conditions for the subsequent governments and thereby alter permanently the convergence towards a new steady state. Said differently, Figure 7, Panel A suggests that a short-lived increase in the perceived probability of war or risk aversion over the war state has the potential to induce a peaceful hegemonic transition, and subsequent continued military and financial strengthening of the newly dominant country.

Figure 7, Panel B, shows that both the fragility and multiplicity thresholds increase with the natural real rate, and that the multiplicity threshold declines strongly with the military depreciation rate, δ . The intuition is that a lower natural real rate lowers borrowing costs across the board, increasing bond market revenue and thereby enhancing the feedback channel from the safety of a country's bonds to how much it can invest in military capacity. The military depreciation rate, shown on the y-axis, strongly decreases the multiplicity threshold, but not the fragility threshold. When the military depreciation rate is high, multiple steady states emerge only at a higher level of debt capacity, but then become immediately fragile because both countries' military capacity gets reset close to zero every period.³³ These results hence suggest that the long decline in the natural

³²See also [Rietz \(1988\)](#), [Barro and Ursua \(2008\)](#), [Gabaix \(2012\)](#), [Wachter \(2013\)](#).

³³The fragility threshold depicted in Panel B is independent of military depreciation, since the existence of a zone

real rate over the previous decades can enhance multiplicity and fragility. Since military capital can depreciate more quickly in times of rapid military technological progress, our model suggests that hysteresis is less likely during such times and that geopolitical fragility becomes more relevant.

Figure 8 shows numerically that the multiplicity and fragility thresholds decline, i.e. multiplicity and fragility become possible for a larger region of the parameter space, when the second country catches up in borrowing capacity (Panel A) and in real GDP (Panel B). Panel A shows that the multiplicity threshold shrinks by a factor of two when both countries have identical borrowing capacity compared to when country 2 has only 10% of country 1’s borrowing capacity. Panel B shows that the multiplicity threshold shrinks by a factor of three as the relative tax base goes from $\tau_2/\tau_1 = 30\%$ to $\tau_2/\tau_1 = 65\%$, roughly China’s GDP as a fraction of the U.S. in 2008 vs. 2023.³⁴ Overall, these numerical results suggest that more similar country sizes and borrowing capacities are quantitatively important for multiplicity and fragility to emerge.

6 Discussion

Our theoretical framework offers a powerful lens to analyze the conditions under which hegemonic transitions occur, both historically and in the present. Even when hegemonic powers engage in major wars, such conflicts do not necessarily result in immediate or inevitable shifts in global leadership. Consider, for example, the case of the U.K. in the 19th century. The Crimean war of 1851 exposed significant military and logistical weaknesses and led to heavy losses for the British. However, despite this significant reduction in military capacity, and the relative ascendancy of the U.S., the U.K. retained its hegemonic role for several more decades. It was not until the far more devastating impact of World War I—when the U.K.’s industrial base was severely diminished, and its fiscal position deteriorated—that the U.S. started to supplant it as the preeminent global power (Kindleberger (1973), Kennedy (1987)).

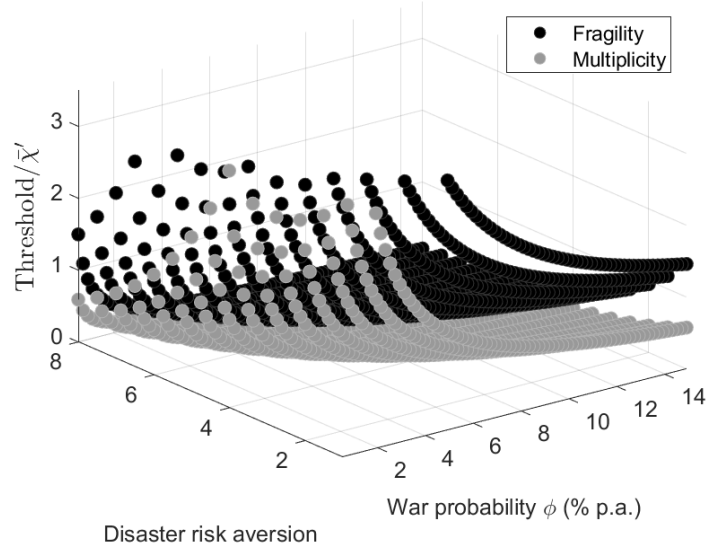
Our model offers a structured approach to understanding the mix of long-run and short-run prerequisites for hegemonic transition. In the long run, a potential successor must possess sufficient fiscal and financial capacity to assume the role of hegemon. This capacity is largely determined by the depth of its sovereign debt markets. Crucially, the required threshold for this capacity is not fixed—it declines when the risk of war is high and investor risk aversion is elevated. The interwar experience of the United States is instructive here: although the U.S. emerged from World War I as the world’s largest creditor nation, it took time to deepen and institutionalize its financial markets to a point where it could offer the global public goods of liquidity and safety in a credible and

of fragility does not depend on the depreciation rate. However, the zone of fragility is wider and more likely to contain the steady state if military depreciation is high.

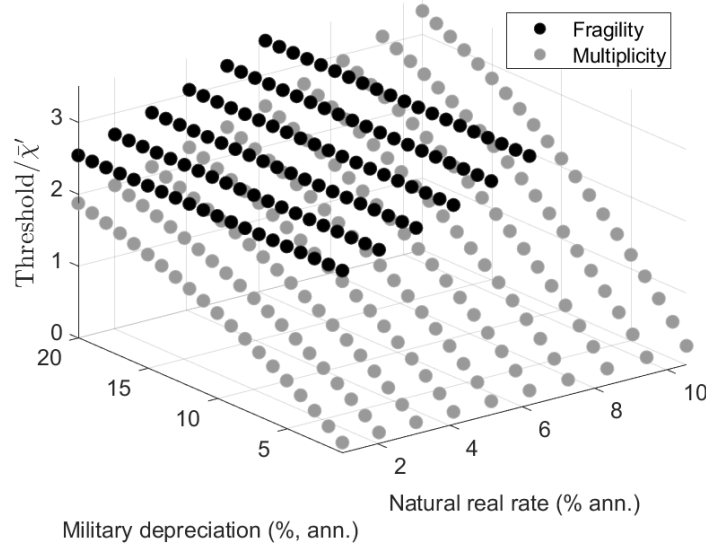
³⁴Solving for the thresholds with asymmetric country size requires taking a stance on the units of military capacity, or equivalent on the effectiveness of one dollar of military spending on the probability of winning a war. We take the effectiveness of military spending on the probability of winning a war from the recent estimates by Federle, Rohner and Schularick (2024), who estimate that a 10% increase in GDP translates into an increase in military spending of 5% of GDP, and increases the probability of winning a war by 3.2 percentage points using commodity shocks. With this, the implied baseline multiplicity threshold with symmetric countries, $\bar{\chi}'$ equals 60% of GDP and the baseline fragility threshold equals 152% of GDP.

Figure 7: Multiplicity and Fragility Thresholds vs. Model Parameters

Panel A: War Probability and Risk Aversion



Panel B: Natural Rate and Military Depreciation

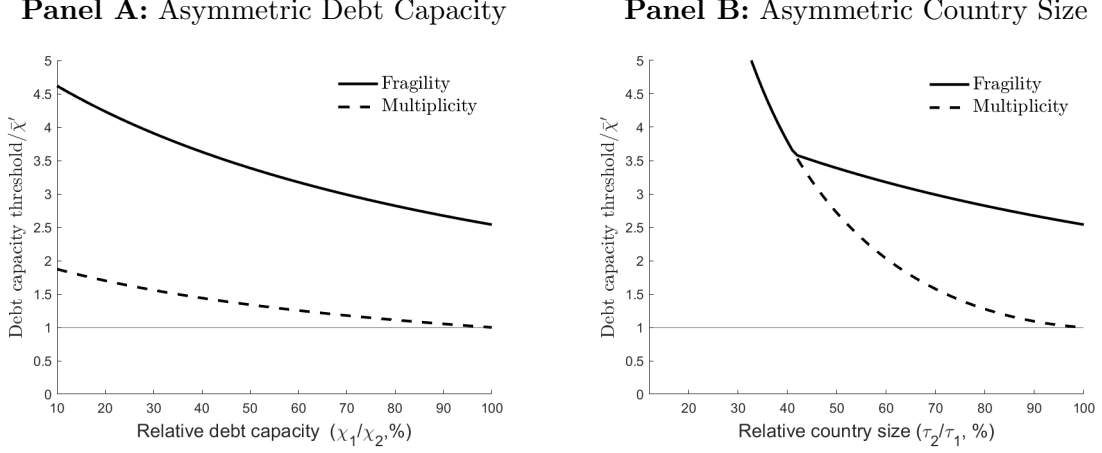


Panel A plots the multiplicity and fragility thresholds in the calibrated model against the investor war risk aversion, ψ , and the war probability, ϕ . Panel B plots the multiplicity and fragility thresholds in the calibrated model against the military depreciation rate, δ , and the natural rate $r^* = -\log \beta$. Only the parameters shown on the axes are varied, while all other parameters are held constant at the baseline calibration values. Both thresholds are relative to the baseline calibration multiplicity threshold, $\bar{\chi}'$.

consistent manner (Eichengreen (2011)).

In terms of short-run dynamics, the model underscores the importance of the war's outcome. If a conflict leaves the incumbent hegemon too weakened to quickly recover—militarily, economically, or institutionally—this can create a window for a rival to assert itself. By contrast, a more limited

Figure 8: Multiplicity and Fragility Thresholds with Asymmetric Countries



Panel A plots the multiplicity and fragility thresholds in the calibrated model for country 1 on the y-axis against the ratio of country 2 to country 1 debt capacity, χ_2/χ_1 , on the x-axis. Panel B plots the multiplicity and fragility thresholds on the y-axis against the ratio of country 2 to country 1 size, τ_2/τ_1 on the x-axis. Only the parameters shown on the x-axis are varied, while all other parameters are held constant at the baseline calibration values and are symmetric for both countries. Multiplicity and fragility thresholds are relative to the baseline calibration multiplicity threshold, $\bar{\chi}'$.

military engagement that does not materially impair the hegemon's core capacities is unlikely to produce regime change without the help of financial markets. However, even a more limited military engagement can potentially act as a sunspot, coordinating bond expectations around a self-fulfilling hegemonic transition if the right long-run conditions are satisfied. Additionally, the rate at which military capital depreciates plays a subtle but important role. A slow depreciation rate means that the military might of the declining hegemon remains formidable for longer, increasing the cost and risk for any challenger seeking to step into its place.

Turning to contemporary geopolitics, our framework provides a structured way to assess whether China could one day displace the United States as the world's dominant power. Of course, this would occur if China develops an insurmountable advantage in military technology. However, in the absence of such a development, a necessary—though not sufficient—condition is the deepening of China's financial markets. While China continues to maintain substantial capital controls, it has also pursued a deliberate strategy of renminbi (RMB) internationalization (Clayton et al. (2022)). Notably, the establishment of the Cross-Border Interbank Payment System (CIPS), bilateral currency swap agreements, and the introduction of a central bank digital currency represent bids to expand the RMB's use in global trade and finance (Bahaj and Reis (2020, 2022), Duffie and Economy (2022)). These developments, though still modest in scale compared to the U.S. dollar's dominance, indicate a growing foundation for a financial infrastructure that could support a future hegemonic role.

Meanwhile, developments within the U.S. raise important questions about its long-term fiscal sustainability. The bond market turmoil of late 2024 and spring 2025—attributed in part to the

actions of so-called “bond market vigilantes”—has amplified concerns over the credibility of U.S. fiscal policy. Should these pressures force a shift toward austerity, our model suggests this could trigger a reinforcing cycle of declining military expenditures, reduced strategic influence, and higher sovereign borrowing costs. Together, these trends could create space for an eventual hegemonic transition.

Importantly, our model does not prescribe that such a transition must be violent. When the perceived risk of war is high and investor risk premiums are elevated, it is possible for a peaceful transition of hegemonic power to occur, especially if global financial market participants can freely exchange assets issued by each potential hegemon, in this case in both U.S. dollars and RMB. In this context, the framework lends credence to the notion that the U.S.–China relationship may lead to a hegemonic transition even in the absence of violent confrontation.

7 Conclusion

We have presented a simple dynamic model in which geopolitics and international bond markets interact. The model is consistent with the three empirical facts: militarily stronger countries have a funding advantage, this advantage rises with geopolitical tensions, and war losers experience higher devaluation than victors.

A key insight of our model is that financial market factors such as greater debt capacity and a higher war premium act as a double-edged sword. They can amplify the stronger country’s military and financial dominance. However, this complementarity also gives rise to multiple steady states as well as a geopolitical fragility, whereby even the exogenously stronger country can lose its hegemonic status if the weaker country is supported by financial markets. This insight can explain Great Britain’s rise to power at the beginning of the nineteenth century. And it highlights the risk that the U.S. faces today if China’s debt capacity rises through the internationalization of its currency, or if the U.S.’s debt capacity becomes imperiled by government policies that erode it. Our model also shows that while hegemonic transitions can occur in the aftermath of wars, peaceful bond market-led transitions from one military and financial hegemon to another are also possible when both countries have very high debt capacities.

Our analysis opens up several interesting directions for future work at the intersection of international finance and conflict. While we have verified the robustness of our results to an endogenous war probability, a fuller analysis how a country’s borrowing decisions interact with the strategic decision to go to war would likely lead to richer insights. Finally, monetary-economic unions and military coalitions could lead to further interactions between multiple countries and bond markets.

References

- Acemoglu, Daron, and Alexander Wolitzky. 2011. “The Economics of Labor Coercion.” *Econometrica*, 79(2): 555–600.
- Acemoglu, Daron, Mikhail Golosov, Aleh Tsyvinski, and Pierre Yared. 2012. “A Dynamic Theory of Resource Wars.” *Quarterly Journal of Economics*, 127(1): 283–331.
- Aguiar, Mark, Manuel Amador, Emmanuel Farhi, and Gita Gopinath. 2015. “Coordination and Crisis in Monetary Unions.” *Quarterly Journal of Economics*, 130(4): 1727–1779.
- Alesina, Alberto, and Andrea Passalacqua. 2016. “Chapter 33 - The Political Economy of Government Debt.” *Handbook of Macroeconomics*, 2: 2599–2651.
- Alesina, Alberto, and Guido Tabellini. 1990. “A Positive Theory of Fiscal Deficits and Government Debt.” *Review of Economic Studies*, 57(3): 403–414.
- Atkeson, Andrew, Jonathan Heathcote, and Fabrizio Perri. 2022. “The End of Privilege: A Reexamination of the Net Foreign Asset Position of the United States.” National Bureau of Economic Research Working paper wp29771.
- Azzimonti, Marina, Eva de Francisco, and Vincenzo Quadrini. 2014. “Financial Globalization, Inequality, and the Rising Public Debt.” *American Economic Review*, 104(8): 2267–2302.
- Bahaj, Saleem, and Ricardo Reis. 2020. “Jumpstarting an International Currency.” *working paper, London School of Economics and University College London*.
- Bahaj, Saleem, and Ricardo Reis. 2022. “Central bank swap lines: Evidence on the effects of the lender of last resort.” *Review of Economic Studies*, 89(4): 1654–1693.
- Baliga, Sandeep, and Tomas Sjöström. 2004. “Arms Races and Negotiations.” *Review of Economic Studies*, 71(2): 351–369.
- Barro, Robert J. 1974. “Are Government Bonds Net Wealth?” *Journal of Political Economy*, 82(6): 1095–1117.
- Barro, Robert J. 2006. “Rare Disasters and Asset Markets in the Twentieth Century.” *Quarterly Journal of Economics*, 121(3): 823–866.
- Barro, Robert J., and Jose F. Ursua. 2008. “Consumption Disasters in the Twentieth Century.” *American Economic Review*, 98(2): 58–63.
- Becko, John S., and Daniel O’Connor. 2024. “Strategic (Dis)Integration.” *Working Paper*.
- Bordo, Michael D., and Eugene N. White. 1991. “A Tale of Two Currencies: British and French Finance during the Napoleonic Wars.” *The Journal of Economic History*, 51(2): 303–316.
- Broner, Fernando, Alberto Martin, Josephin Meyer, and Christoph Trebesch. 2024. “Hegemonic Globalization.” *Working Paper*.
- Caldara, Dario, and Matteo Iacoviello. 2022. “Measuring Geopolitical Risk.” *American Economic Review*, 112(4): 1194–1225.
- Chahrour, Ryan, and Rosen Valchev. 2022. “Trade Finance and the Durability of the Dollar.” *Review of Economic Studies*, 89(4): 1873–1910.
- Chari, V.V., and Patrick J. Kehoe. 2007. “On the Need for Fiscal Constraints in a Monetary Union.” *Journal of Monetary Economics*, 54(8): 2399–2408.
- Chen, Zefeng, Zhengyang Jiang, Hanno Lustig, Stijn Van Nieuwerburgh, and Mindy Z. Xiaolan. 2025. “Exorbitant Privilege Gained and Lost: Fiscal Implications.” *Journal of Political Economy*, forthcoming.
- Choi, Jason, Rishabh Kirpalani, and Diego Perez. 2024. “US public debt and safe asset market power.” *Journal of Political Economy*.
- Clark, Gregory. 1996. “The political foundations of modern economic growth: England, 1540–1800.” *Journal of Interdisciplinary History*, 26(4): 563–588.
- Clayton, Christopher, Amanda Dos Santos, Matteo Maggiori, and Jesse Schreger. 2022.

- “Internationalizing Like China.” *National Bureau of Economic Research working paper wp30336*.
- Clayton, Christopher, Amanda Dos Santos, Matteo Maggiori, and Jesse Schreger.** 2025. “International Currency Competition.” *NBER Working Paper wp31975*.
- Clayton, Christopher, Matteo Maggiori, and Jesse Schreger.** 2023. “A Framework for Geoeconomics.” *National Bureau of Economic Research working paper wp31852*.
- Cole, Harold L, and Timothy J Kehoe.** 2000. “Self-fulfilling debt crises.” *The Review of Economic Studies*, 67(1): 91–116.
- Cooley, Thomas F, and Edward C Prescott.** 1995. “Economic growth and business cycles.” *Frontiers of business cycle research*, 1: 1–38.
- Coppola, Antonio, Arvind Krishnamurthy, and Chenzi Xu.** 2023. “Liquidity, Debt Denomination, and Currency Dominance.” *National Bureau of Economic Research working paper wp30984*.
- Diehl, Paul F, and Jean Kingston.** 1987. “Messenger or message? Military buildups and the initiation of conflict.” *Journal of Politics*, 49(3): 801–813.
- Dovis, Alessandro, and Rishabh Kirpalani.** 2020. “Fiscal rules, bailouts, and reputation in federal governments.” *American Economic Review*, 110(3): 860–888.
- Duffie, Darrell, and Elizabeth Economy,** ed. 2022. *Digital Currencies: The US, China, and the World at a Crossroads*. Stanford, CA:Hoover Institution Press.
- Du, Wenxin, Carolin E. Pflueger, and Jesse Schreger.** 2020. “Sovereign debt portfolios, bond risks, and the credibility of monetary policy.” *Journal of Finance*, 75(6): 3097–3138.
- Eichengreen, Barry.** 2011. *Exorbitant Privilege: The Rise and Fall of the Dollar and the Future of the International Monetary System*. Oxford University Press.
- Farhi, Emmanuel, and Matteo Maggiori.** 2018. “A Model of the International Monetary System.” *The Quarterly Journal of Economics*, 133(1): 295–355.
- Fearon, James D.** 1995. “Rationalist Explanations for War.” *International Organization*, 49(3): 379–414.
- Federle, Jonathan, Dominic Rohner, and Moritz Schularick.** 2024. “Who wins wars?” *Available at SSRN 5093393*.
- Federle, Jonathan, Johannes Marzian, and Christoph Trebesch.** 2024. “Guns vs. Butter, 1870–2022.” *Kiel Institute Working Paper*.
- Ferguson, Niall.** 2006. “Political Risk and the International Bond Market between the 1848 Revolution and the Outbreak of the First World War.” *Economic History Review*, 59(1): 70–112.
- Ferguson, Niall.** 2008. *The Ascent of Money: A Financial History of the World*. Penguin.
- Fernández-Villaverde, Jesús, Tomohide Mineyama, and Dongho Song.** 2024. “Are We Fragmented Yet? Measuring Geopolitical Fragmentation and its Causal Effects.” *Working paper, University of Pennsylvania, Boston College, and Johns Hopkins*.
- Fornaro, Luca, and Christoph Grosse-Steffen.** 2025. “Fragmented monetary unions.” *Working Paper, Universitat Pompeu Fabra and Banque de France*.
- Fudenberg, Drew, and Jean Tirole.** 1991. *Game Theory*. MIT press.
- Gabaix, Xavier.** 2012. “Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance.” *The Quarterly journal of economics*, 127(2): 645–700.
- Gennaioli, Nicola, and Hans-Joachim Voth.** 2015. “State Capacity and Military Conflict.” *The Review of Economic Studies*, 82(4): 1409–1448.
- Gopinath, Gita, and Jeremy C Stein.** 2021. “Banking, Trade, and the Making of a Dominant Currency.” *Quarterly Journal of Economics*, 136(2): 783–830.
- Gorodnichenko, Yuriy, Iikka Korhonen, and Elina Ribakova.** 2024. “Russian economy on war footing: A new reality financed by commodity exports.”
- Gourinchas, Pierre-Olivier, and Helene Rey.** 2022. “Exorbitant Privilege and Exorbitant

- Duty.”
- Halac, Marina, and Pierre Yared.** 2018. “Fiscal Rules and Discretion in a World Economy.” *American Economic Review*, 108(8): 2305–2334.
- Hassan, Tarek A.** 2013. “Country Size, Currency Unions, and International Asset Returns.” *Journal of Finance*, 68(6): 2269–2308.
- He, Zhiguo, Arvind Krishnamurthy, and Konstantin Milbradt.** 2019. “A Model of Safe Asset Determination.” *American Economic Review*, 109(4): 1230–1262.
- Hirschman, Albert O.** 1945. *National Power and the Structure of Foreign Trade*. Publication, University of California Press.
- Hirschman, Albert O.** 1958. *The Strategy of Economic Development*. Yale Studies in Economics, Yale University Press.
- Hirshleifer, David, Dat Mai, and Kuntara Pukthuanthong.** 2025. “War Discourse and Disaster Premium: 160 Years of Evidence from the Stock Market.” *Review of Financial Studies*, 38(2): 457–506.
- Hirshleifer, Jack.** 1989. “Conflict and Rent-Seeking Success Functions: Ratio vs. Difference Models of Relative Success.” *Public Choice*, 63(2): 101–112.
- Ilzetzki, Ethan, Carmen M. Reinhart, and Kenneth S Rogoff.** 2022. “Rethinking Exchange Rate Regimes.” In *Handbook of International Economics*. Vol. 6, 91–145. Elsevier.
- Kelly, Bryan, Lubos Pastor, and Pietro Veronesi.** 2016. “The Price of Political Uncertainty: Theory and Evidence from the Option Market.” *Journal of Finance*, 71(5): 2417–2480.
- Kennedy, P.** 1987. *The Rise and Fall of the Great Powers: Economic Change and Military Conflict from 1500 to 2000*. Vintage, Lexington Publishers, Lexington, MA.
- Kindleberger, Charles P.** 1973. *The World in Depression, 1929–1939*. Berkeley:University of California Press.
- Koijen, Ralph S.J., Tomas J. Philipson, and Harald Uhlig.** 2016. “Financial Health Economics.” *Econometrica*, 84(1): 195–242.
- Korevaar, Matthijs, Marc Francke, and Piet Eichholtz.** 2020. “Dure Huizen Maar Geen Zeepbel in Amsterdam.” *Economisch Statistische Berichten*, , (106): 32–34.
- König, Michael D., Dominic Rohner, Mathias Thoenig, and Fabrizio Zilibotti.** 2017. “Networks in Conflict: Theory and Evidence from the Great War of Africa.” *Econometrica*, 85(4): 1093–1132.
- Liu, Ernest, and David Yang.** 2024. “International Power.” *Working Paper*.
- Maggi, Giovanni.** 2016. “Issue Linkage.” *Handbook of Commercial Policy*, 1: 513–564.
- McCandless, George T.** 1996. “Money, expectations, and the US Civil War.” *American Economic Review*, 86(3): 661–671.
- Meissner, Christopher M, and Alan M Taylor.** 2006. “Losing our marbles in the new century? The great rebalancing in historical perspective.”
- Meyer, Josefin, Carmen M. Reinhart, and Christoph Trebesch.** 2022. “Sovereign Bonds since Waterloo.” *Quarterly Journal of Economics*, 137(3): 1615–1680.
- Mohr, Cathrin, and Christoph Trebesch.** 2024. “Goeconomics.”
- North, Douglass C., and Barry R. Weingast.** 1989. “Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England.” *Journal of Economic History*, 49(4): 803–832.
- Padró i Miquel, Gerard, and Pierre Yared.** 2012. “The Political Economy of Indirect Control.” *Quarterly Journal of Economics*, 127(2): 947–1015.
- Pastor, Lubos, and Pietro Veronesi.** 2012. “Uncertainty about Government Policy and Stock Prices.” *Journal of Finance*, 67(4): 1219–1264.
- Pastor, Lubos, and Pietro Veronesi.** 2020. “Political Cycles and Stock Returns.” *Journal of*

- Political Economy*, 128(11): 4011–4045.
- Persson, Torsten, and Lars E. O. Svensson.** 1989. “Why a Stubborn Conservative Would Run a Deficit: Policy with Time-Inconsistent Preferences.” *Quarterly Journal of Economics*, 104(2): 325–345.
- Powell, Robert.** 1993. “Guns, Butter, and Anarchy.” *American Political Science Review*, 87(1): 115–132.
- Powell, Robert.** 1999. *In the Shadow of Power: States and Strategies in International Politics*. Princeton University Press.
- Reinhart, Carmen M., and Kenneth S. Rogoff.** 2009. *This Time is Different: Eight Centuries of Financial Folly*. Princeton, NJ: Princeton University Press.
- Rietz, Thomas A.** 1988. “The equity risk premium a solution.” *Journal of Monetary Economics*, 22(1): 117–131.
- Rigobon, Roberto, and Brian Sack.** 2005. “The Effects of War Risk on US Financial Markets.” *Journal of Banking Finance*, 29(7): 1769–1789.
- Rodger, Nicholas A.M.** 2006. *The Command of the Ocean: A Naval History of Britain 1649–1815*. Penguin UK.
- Sargent, Thomas J, and Francois R. Velde.** 1995. “Macroeconomic Features of the French Revolution.” *Journal of Political Economy*, 103(3): 474–518.
- Schama, Simon.** 1988. *The Embarrassment of Riches: An Interpretation of Dutch Culture in the Golden Age*. Univ of California Press.
- Schmelzing, Paul.** 2020. “Eight Centuries of Global Real Interest Rates, R-G, and the ‘Suprasecular’ Decline, 1311–2018.” *Bank of England Staff Working Paper No. 845*, 1311–2018.
- Skaperdas, Stergios.** 1992. “Cooperation, Conflict, and Power in the Absence of Property Rights.” *American Economic Review*, 82(4): 720–739.
- Stasavage, David.** 2007. “Partisan politics and public debt: The importance of the ‘Whig Supremacy’ for Britain’s financial revolution.” *European Review of Economic History*, 11(1): 123–153.
- Thoenig, Mathias.** 2024. “Trade policy in the shadow of war: A quantitative toolkit for geoeconomics.” *CEPR Working Paper 18419*.
- Wachter, Jessica A.** 2013. “Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?” *Journal of Finance*, 68(3): 987–1035.
- Wallace, Michael D.** 1979. “Arms races and escalation: Some new evidence.” *Journal of Conflict Resolution*, 23(1): 3–16.
- Yared, Pierre.** 2019. “Rising Government Debt: Causes and Solutions for a Decades-Old Trend.” *Journal of Economic Perspectives*, 33(2): 115–140.

Appendix

A Proofs

A.1 Proof of Proposition 1

We define

$$\chi'(A, \chi, \phi, \tau, \theta) = \frac{\tilde{h}^{-1}(A)}{\phi\tau\theta}, \quad (\text{A.1})$$

where \tilde{h}^{-1} is the inverse function of

$$\tilde{h} : [2, \infty) \mapsto [0, \infty), \quad (\text{A.2})$$

$$\tilde{h}(x) = -2 \tanh^{-1} \left(\sqrt{1 - \frac{2}{x}} \right) + x \sqrt{1 - \frac{2}{x}}. \quad (\text{A.3})$$

Here $\tanh : (-\infty, \infty) \mapsto (-1, 1)$ is the hyperbolic tangent function. Differentiating \tilde{h} gives

$$\frac{d\tilde{h}(x)}{dx} = \sqrt{1 - \frac{2}{x}} \quad \forall x \in [2, \infty). \quad (\text{A.4})$$

Here, we used that $\frac{d}{dz} \tanh(z) = 1 - \tanh^2(z)$. This implies that \tilde{h} is a strictly increasing function $[2, \infty) \mapsto [0, \infty)$, so its inverse $\tilde{h}^{-1} : [0, \infty) \mapsto [2, \infty)$ exists and is also strictly increasing. We therefore have that $\chi'(A, \chi, \phi, \tau, \theta) \geq \frac{2}{\phi\tau\theta}$, $\frac{d\chi'}{dA} > 0$, $\frac{d\chi'}{d\phi} < 0$, $\frac{d\chi'}{d\tau} < 0$, and $\frac{d\chi'}{d\theta} < 0$.

Now suppose that $\chi < \chi'$, i.e. we are in case i). Rearranging equation (5) and defining the effective difference in military strength $z = A + m_1 - m_2$, any equilibrium must be a zero of the function

$$h(z) = -A + z - \chi\phi\tau\theta (2F(z) - 1) = -A + z - \chi\phi\tau\theta \tanh\left(\frac{z}{2}\right), \quad (\text{A.5})$$

The derivative of h with respect to z is

$$\frac{dh(z)}{dz} = 1 - \frac{\chi\phi\tau\theta}{2} \left(1 - \tanh^2\left(\frac{z}{2}\right) \right) \quad (\text{A.6})$$

Since $-1 < \tanh(z) < 1$ and $\tanh(z) \xrightarrow{z \rightarrow \pm\infty} \pm 1$, it is clear that $h(z)$ goes from $-\infty$ to ∞ . There are two sub-cases to case i). If χ is sufficiently low that $\chi \leq \chi'(0, \chi, \phi, \tau, \theta) = \frac{2}{\phi\tau\theta}$, then $0 < \frac{dh(z)}{dz} < 1$ almost everywhere and h is strictly increasing, showing that the equilibrium is unique.

Next, consider the sub-case where $\chi < \chi'$ but $\chi > \frac{2}{\phi\tau\theta}$. Local maxima and minima are roots of $\frac{dh}{dz}(z) = 0$

$$z_{\pm} = \pm 2 \tanh^{-1} \left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}} \right). \quad (\text{A.7})$$

Hence, there exist exactly two local extrema. Because h goes from $-\infty$ to ∞ , the lower root, z_- , is a local maximum and the upper root, z_+ , is a local minimum. Substituting for z_- and using that

$\tanh(-z) = -\tanh(z)$ the height of the local maximum is

$$h(z_-) = -A + \underbrace{\left(-2 \tanh^{-1} \left(\sqrt{1 - \frac{2}{\chi\phi\tau\theta}} \right) + \chi\phi\tau\theta \sqrt{1 - \frac{2}{\chi\phi\tau\theta}} \right)}_{\tilde{h}(\chi\phi\tau\theta)}. \quad (\text{A.8})$$

Because $\chi < \chi'$ the local maximum is negative, implying that h crosses zero only once and the equilibrium is unique.

If in addition $A = 0$ then $h(0) = 0$, implying that the unique equilibrium is given by $z^* = 0$. Conversely, if $A > 0$, then $h(A) < 0$, and since $h \xrightarrow{z \rightarrow \infty} \infty$ the unique zero satisfies $z^* > A$, implying that $w_1(m_1^*, m_2^*) > \frac{1}{2}$, $m_1^* > m_2^*$ and $q_1^* > q_2^*$.

Now suppose that $\chi > \chi'$, i.e. we are in case ii). Equation (A.8) then implies that $h(z_-) > 0$. Because country 1 is assumed to have a weak exogenous military advantage, i.e. $A \geq 0$, we have $h(0) \leq 0$. With $z_- < 0$, h going from minus to plus infinity, and h having exactly one local maximum at z_- and one local minimum at z_+ it follows that h crosses zero exactly three times, and there are three equilibria.

Label the equilibria $z^{1,*} < z^{2,*} < z^{3,*}$. Since z_- is a local maximum and z_+ is a local minimum of h , we have that $z^{1,*} < -z_- < 0$ and $z^{3,*} > z_+ > 0$ implying that $w_1(m_1^*(z^{1,*}), m_2^*(z^{1,*})) = \frac{1}{2} \tanh\left(\frac{z^{1,*}}{2}\right) + \frac{1}{2} < \frac{1}{2}$ and $w_1(m_1^*(z^{3,*}), m_2^*(z^{3,*})) = \frac{1}{2} \tanh\left(\frac{z^{3,*}}{2}\right) + \frac{1}{2} > \frac{1}{2}$. With $w_2(m_1^*, m_2^*) = 1 - w_1(m_1^*, m_2^*)$ and the bond pricing equation (2), the remaining inequalities follow.

Employing an asymptotic stability concept, where the bond market sets prices in alternation with governments choosing military investment, an equilibrium is stable if and only if the slope of the bond revenue function at the equilibrium is below one in absolute value. Because $\tanh(\frac{z^{1,*}}{2}) < \tanh(\frac{z_-}{2}) < \tanh(\frac{z^{2,*}}{2}) < \tanh(\frac{z_+}{2}) < \tanh(\frac{z^{3,*}}{2})$, and z_{\pm} are zeros of (A.6) and \tanh is between zero and one, it follows that $\frac{\chi\phi\tau\theta}{2} \left(1 - \tanh^2\left(\frac{z^{1,*}+A}{2}\right)\right) < 1$, $\frac{\chi\phi\tau\theta}{2} \left(1 - \tanh^2\left(\frac{z^{3,*}+A}{2}\right)\right) < 1$ and $\frac{\chi\phi\tau\theta}{2} \left(1 - \tanh^2\left(\frac{z^{2,*}+A}{2}\right)\right) > 1$, i.e. the slope of the bond revenue function with respect to $m_1 - m_2$ is greater than one at $z^{2,*}$, but less than one at $z^{1,*}$ and $z^{3,*}$. ■

A.2 Proof of Corollary 1

Suppose that country 1 is dominant in equilibrium, meaning that $w_1(m_1^*, m_2^*) > \frac{1}{2}$. Further assume that the equilibrium is stable. We start by proving the statement for θ . Note that $\frac{dw_1^*}{dz^*} > 0$ from equation (1) and $\frac{d(q_1^* - q_2^*)}{dw_1^*} > 0$ from equation (2), so it is sufficient to prove that $\frac{dz^*}{d\theta} > 0$. We totally differentiate the equilibrium condition $h(z^*) = 0$ with respect to θ and use the implicit function theorem:

$$\frac{dz^*}{d\theta} = \frac{\chi\phi\tau \tanh\left(\frac{z^*}{2}\right)}{1 - \frac{1}{2}\chi\phi\tau\theta \left(1 - \tanh^2\left(\frac{z^*}{2}\right)\right)}. \quad (\text{A.9})$$

The denominator in this expression is positive because we are in a stable equilibrium and the numerator is positive because country 1 is dominant, implying that $\tanh\left(\frac{z^*}{2}\right) = 2(w_1(m_1^*, m_2^*) - 1) > 0$. If country 2 is dominant in equilibrium, the sign of the numerator in (A.9) is negative, and $\frac{dz^*}{d\theta} < 0$, implying $\frac{dw_2^*}{dz^*} > 0$ and $\frac{d(q_2^* - q_1^*)}{dz^*} > 0$. The proofs for χ and ϕ are analogous.

Finally, totally differentiating the equilibrium condition $h(z^*)$ with respect to the exogenous

military advantage A gives

$$\frac{dz^*}{dA} = \frac{1}{1 - \frac{1}{2}\chi\phi\tau\theta(1 - \tanh^2(\frac{z^*}{2}))} > 0, \quad (\text{A.10})$$

Because the military advantage of country 2 equals $-A$, this proves the comparative static with respect to the exogenous military advantage for both countries. ■

A.3 Proof of Proposition 2

Define the threshold

$$\chi'(A, \chi, \phi, \tau, \theta, \delta) = \frac{\delta\tilde{h}^{-1}(A)}{\tau\phi\theta}, \quad (\text{A.11})$$

where the function $\tilde{h}[0, \infty) \mapsto [2, \infty)$ is defined as in Appendix Section A.1. Because \tilde{h} is strictly increasing, it is clear that $\chi' \geq \frac{2\delta}{\phi\tau\theta}$, $\frac{d\chi'}{dA} > 0$, $\frac{d\chi'}{d\phi} < 0$, $\frac{d\chi'}{d\tau} < 0$, $\frac{d\chi'}{d\theta} < 0$, $\frac{d\chi'}{d\delta} > 0$.

With an abuse of notation, a steady state is a zero of the function

$$h(z) = \delta(z - A) - \tau\chi\phi\theta \tanh\left(\frac{z}{2}\right). \quad (\text{A.12})$$

The remainder of the proof for cases i) and ii) is analogous to Appendix Section A.1. ■

A.4 Proof of Proposition 3

Define the threshold

$$\chi'' = \frac{2}{\tau\phi\theta}. \quad (\text{A.13})$$

The threshold χ'' has the following properties: $\frac{d\chi''}{d\phi} < 0$, $\frac{d\chi''}{d\theta} < 0$, $\frac{d\chi''}{d\tau} < 0$.

Suppose we are in case i), i.e. $\chi < \chi''$. Define again $z_t = \mu_t + A$ as the effective relative military capacity of country 1. The equilibrium mapping (8) can be written as

$$z_{t-1} = H(z_t), \quad (\text{A.14})$$

$$H(z_t) \equiv \frac{z_t - \tau\chi\phi\theta \tanh\left(\frac{z_t}{2}\right) - \delta A}{1 - \delta}. \quad (\text{A.15})$$

Taking the derivative

$$\frac{dH(z)}{dz} = \frac{1 - \frac{\tau\chi\phi\theta}{2}(1 - \tanh^2(\frac{z}{2}))}{1 - \delta} \quad (\text{A.16})$$

Because $\chi < \chi''$ and \tanh is bounded between -1 and 1 , it follows that H is strictly increasing and hence one-to-one. As a result, any transition path must be monotonic and the steady-state is uniquely determined by the initial conditions.

Now, suppose we are in case ii). In that case, $\frac{dH(z)}{dz}(z) < 0$ for all $z \in (\underline{z}, \bar{z})$, where

$$\underline{z} = -2 \tanh^{-1} \left(\sqrt{1 - \frac{2}{\chi \phi \tau \theta}} \right) \quad (\text{A.17})$$

$$\bar{z} = 2 \tanh^{-1} \left(\sqrt{1 - \frac{2}{\chi \phi \tau \theta}} \right). \quad (\text{A.18})$$

For any $z_{t-1} \in (H(\underline{z}), H(\bar{z}))$ there exist multiple z_t such that (A.14) is satisfied, and we call this interval the zone of fragility. Hence, there exists at least one z_{t-1} such that z_t could take multiple values.

Next, we show that there exists a z_{t-1} such that z_t can take multiple values and $z_t - z_{t-1}$ can take either sign. To prove this, we show that the middle steady state lies in the zone of fragility. The local extrema of $h(z)$, defined in equation (A.12), are given by

$$z_{\pm} = \pm 2 \tanh^{-1} \left(\sqrt{1 - \frac{2\delta}{\chi \phi \tau \theta}} \right). \quad (\text{A.19})$$

Because the middle steady state, $z^{2,*}$, satisfies $z_- < z^{2,*} < z_+$ and $\delta < 1$ it follows that $\underline{z} < z^{2,*} < \bar{z}$ and $H(\bar{z}) > H(z^{2,*}) = z^{2,*} > H(\underline{z})$. This proves that the middle steady state lies in the zone of fragility.

We next show that there exists a non-monotonic transition path within an arbitrarily small neighborhood of $z^{2,*}$. We know that there exists $\epsilon > 0$ such that $(z^{2,*} - \epsilon, z^{2,*} + \epsilon)$ lies entirely in the zone of fragility. Since H intersects the 45-degree line at $z^{2,*}$ it follows that there exists a z_{t-1} within this neighborhood such that $z_t - z^{2,*}$ and $z_t - z_{t-1}$ can take either sign. Hence, there exists at least one non-monotonic transition path and transition paths need not be unique.

Since $h'(z^{2,*}) < 0$ we know that $\frac{H(z^{2,*})}{dz} < 1$. If $-1 < \frac{H(z^{2,*})}{dz} < 1$, the middle steady state $z^{2,*}$ is unstable. If z_t jumps below $z^{2,*}$, there exists a convergence path to $z^{3,*}$ and for $z_t < z^{2,*}$ there exists a convergence path to $z^{1,*}$.

If $\frac{H(z^{2,*})}{dz} < -1$, the middle steady state is stable but transition paths converging to it are non-monotonic. Because H^{-1} is downward-sloping at $z^{2,*}$, from within a neighborhood of $z^{2,*}$, it is hence possible to jump to three different values for z_t , one of which is on the upper portion (\bar{z}, ∞) and one of which is on the lower portion $(-\infty, \underline{z})$ of the curved line in Figure 5, Panel C. If $z_t \in (\bar{z}, \infty)$ there exists a transition path leading to $z^{3,*}$, while if $z_t \in (-\infty, \underline{z})$ there exists a transition path to $z^{1,*}$. This proves that there exists a non-monotonic transition path to each of the monotonically stable steady states. ■

A.5 Proof of Corollary 2

Corollary 2 follows from Proposition 3. Only the statement about discount rates in ii) requires an additional proof. Because we want to take the limit as $\delta \rightarrow 1$ while ensuring that $\chi > \chi'$ and $\chi > \chi''$ throughout, assume that $\chi > \frac{\tilde{h}^{-1}(A)}{\tau \phi \theta} \geq \chi''$. By continuity, as $\delta \rightarrow 1$ the zeros of h defined in (A.12) converge to those of $h^{\delta=1}$. This shows that $z^{1,*}$ and $z^{3,*}$ are bounded as $\delta \rightarrow 1$. Conversely, the region of fragility equals

$$(H(\underline{z}), H(\bar{z})) = \left(-\frac{\delta A}{1-\delta} - \frac{\tilde{h}(\chi \phi \tau \theta)}{1-\delta}, -\frac{\delta A}{1-\delta} + \frac{\tilde{h}(\chi \phi \tau \theta)}{1-\delta} \right), \quad (\text{A.20})$$

whose left bound goes to $-\infty$ and the right bound goes to ∞ . This shows that for δ sufficiently close to one, all three steady states lie within the region of fragility. ■

A.6 Proofs of Proposition 4

A steady state is a zero of the function

$$h(z) = \delta z - \left(-\phi + \frac{\phi\theta}{2}\right) \tau (\chi_1 - \chi_2) - \phi\theta\tau \frac{\chi_1 + \chi_2}{2} \tanh\left(\frac{z}{2}\right). \quad (\text{A.21})$$

By analogy to Proposition 2 there exists a unique steady state with country 1 dominating if

$$\tilde{h}\left(\frac{\frac{\chi_1 + \chi_2}{2}\tau\phi\theta}{\delta}\right) < \frac{1}{\delta} \left(-\phi + \frac{\phi\theta}{2}\right) \tau (\chi_1 - \chi_2), \quad (\text{A.22})$$

where $\tilde{h} : [2, \infty) \mapsto [0, \infty)$ is the same strictly increasing function as before. The condition $\chi_1 > \frac{4}{\tau\phi\theta}$ ensures that h has a local maximum and a local minimum for any value of χ_2 and that \tilde{h} is defined on its argument.

Define the threshold η

$$\eta = \frac{\tilde{h}\left(\frac{\frac{\chi_1 + \chi_2}{2}\tau\phi\theta}{\delta}\right)}{\frac{1}{\delta} \left(-\phi + \frac{\phi\theta}{2}\right) \tau}. \quad (\text{A.23})$$

By continuity of \tilde{h} , it then follows that the steady state is unique if $\chi_1 - \chi_2 > \eta$, but if $\chi_1 - \chi_2 < \eta$ there are two monotonically stable and one not monotonically stable steady state. The remainder of the proof for cases i) and ii) is analogous to Appendix Section A.1. ■