Online Appendix for "Sovereign Debt Portfolios, Bond Risks, and the Credibility of Monetary Policy"

(Not for Publication)

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This online appendix consists of Section A, "Empirical Appendix", and Section B, "Model Appendix."

A Empirical Appendix

A.1 Currency names and codes

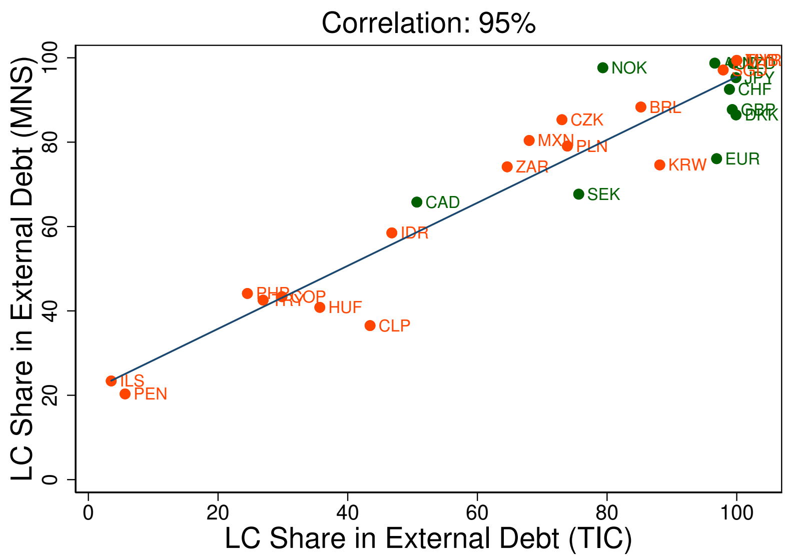
Table AI lists the country name, currency name, and the three-letter currency code for our sample countries.

Table AI: Currency names and codes

Developed markets			Emerging markets		
Country	Currency	Code	Country	Currency	Code
Australia	Australian dollar	AUD	Brazil	Brazilian real	BRL
Canada	Canadian dollar	CAD	Chile	Chilean peso	CLP
Denmark	Danish krone	DKK	Colombia	Colombian peso	COP
Germany	Euro	EUR	Czech Republic	Czech koruna	CZK
Japan	Japanese yen	JPY	Hungary	Hungarian forint	HUF
New Zealand	New Zealand dollar	NZD	Indonesia	Indonesian rupiah	IDR
Norway	Norwegian krone	NOK	Israel	Israeli shekel	ILS
Sweden	Swedish krona	SEK	Malaysia	Malaysian ringgit	MYR
Switzerland	Swiss franc	CHF	Mexico	Mexican peso	MXN
United Kingdom	British pound	GBP	Peru	Peruvian nuevo sol	PEN
United States	US dollar	USD	Philippines	Philippine peso	PHP
			Poland	Polish zloty	PLN
			Singapore	Singapore dollar	SGD
			South Africa	South African rand	ZAR
			South Korea	South Korean won	KRW
			Thailand	Thai baht	THB
			Turkey	Turkish lira	TRY

A.2 Comparing External Debt Sources

Figure A1: External LC Debt Share in Global Mutual Funds and US *TIC*, 2015

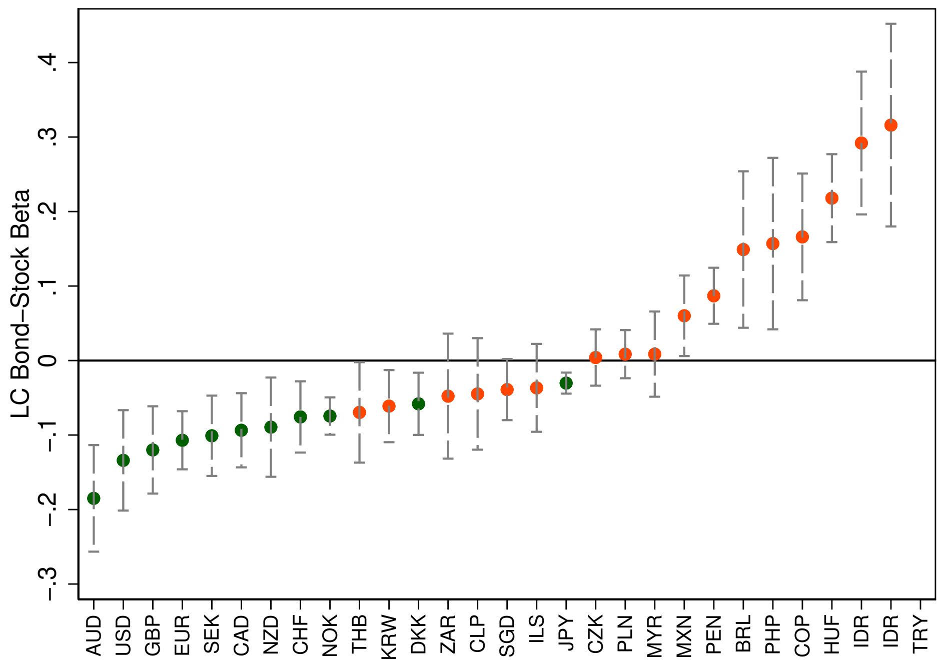


Note: This figure plots the percentage of external government debt denominated in each country's local currency using data from global mutual funds in Maggiori et al. (2019) (MNS) and the US external position in the Treasury International Capital (*TIC*) data. MNS data uses data for the entire European Monetary Union (EMU). *TIC* data uses Germany for the Euro area. All data are for end of year 2015.

A.3 Estimates of LC bond-stock betas by country

Figure A2 plots our LC bond-local stock beta, $\beta(bond_i, stock_i)$, estimated from Eqn. (3) by country. The 95 % confidence intervals are based on the bootstrap described in detail in Section A.3.1. The betas are precisely estimated and most of them are statistically significantly different from zero. Developed markets (shown in green) generally have negative bond-stock betas, and emerging markets (shown in red) have higher bond-stock betas, with many countries having positive bond-stock betas. Even within emerging markets, we document economically and statistically significant cross-country heterogeneity in LC bond-stock betas.

Figure A2: Individual Country LC Bond-Stock Betas



Note: This figure plots the baseline LC bond-stock beta estimated as in Eqn. (3) for each country. The green dots denote point estimates for developed markets. The red dots denote point estimates for emerging markets. The vertical bars denote 95% confidence interval based on bootstrap standard errors. The details of the bootstrap are described in Section A.3.1.

Table AII reports regression estimates for the LC bond-local stock betas by country, $\beta(bond_i, stock_i)$. It shows the same point estimates as in Figure A2, together with Newey-West standard errors with 120-day lags and bootstrap standard errors.

Table AII: Regression Estimates of Individual Country LC Bond-Stock Betas

	LC Bond-Stock Beta	Newey-West SE	Bootstrap SE	N	R^2
AUD	-0.185***	(0.0327)	(0.0365)	2,453	0.406
BRL	0.149**	(0.0512)	(0.0536)	2,044	0.117
CAD	-0.0936***	(0.0218)	(0.0254)	2,421	0.207
CHF	-0.0757***	(0.0207)	(0.0244)	2,421	0.185
CLP	-0.0448	(0.0377)	(0.0382)	2,189	0.025
COP	0.166***	(0.0411)	(0.0434)	2,291	0.215
CZK	0.00405	(0.0154)	(0.0193)	2,420	0.001
DKK	-0.0581***	(0.0172)	(0.0213)	2,395	0.108
EUR	-0.107***	(0.0180)	(0.0199)	2,484	0.302
GBP	-0.120***	(0.0269)	(0.0299)	2,455	0.175
HUF	0.218***	(0.0255)	(0.0301)	2,393	0.387
IDR	0.292***	(0.0372)	(0.0489)	2,291	0.340
ILS	-0.0367	(0.0203)	(0.0301)	1,701	0.033
JPY	-0.0303***	(0.00624)	(0.0072)	2,315	0.185
KRW	-0.0612**	(0.0217)	(0.0247)	2,361	0.102
MXN	0.0601**	(0.0234)	(0.0276)	2,428	0.048
MYR	0.00862	(0.0269)	(0.0292)	2,338	0.002
NOK	-0.0745***	(0.00888)	(0.0128)	2,422	0.268
NZD	-0.0894**	(0.0312)	(0.0340)	2,427	0.080
PEN	0.0869***	(0.0192)	(0.0192)	2,124	0.267
PHP	0.157**	(0.0520)	(0.0587)	2,289	0.138
PLN	0.00857	(0.0143)	(0.0165)	2,402	0.002
SEK	-0.101***	(0.0262)	(0.0275)	2,423	0.220
SGD	-0.0390*	(0.0155)	(0.0209)	2,423	0.071
THB	-0.0696**	(0.0285)	(0.0344)	2,283	0.088
TRY	0.316***	(0.0676)	(0.0694)	2,248	0.296
USD	-0.134***	(0.0207)	(0.0344)	2,427	0.269
ZAR	-0.0478	(0.0345)	(0.0428)	2,394	0.021

Note: This table shows the regression estimates of the LC bond-local stock beta, $\beta(bond_i, stock_i)$, based on Eqn. (3) by country. The regressions are estimated using daily observations on overlapping one-quarter holding returns from 2005 to 2014. Newey-West standard errors are used with 120-day lags to adjust for overlapping holding periods of returns. Bootstrap standard errors are computed as the standard deviation of bond-stock betas estimated on bootstrapped data, $\hat{\beta}^{boot}(bond_i, stock_i)$, where the standard deviation is taken across 500 independent bootstraps. The bootstrap procedure adjusts for serial correlation and heteroskedasticity in bond and stock returns and is described in detail in Appendix A.3.1. Statistical significance is based on the bootstrap standard errors, with the significance level indicated by *** p<0.01, ** p<0.05, * p<0.1.

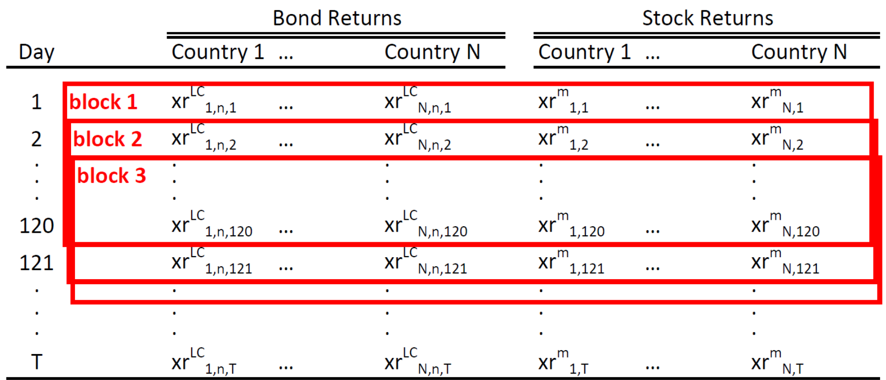
A.3.1 Details: Bootstrap Standard Errors

We now give the implementation details for the bootstrap standard errors shown in Figure A2 and Table AII. We generate bootstrapped bond and stock returns while accounting for serial correlation and heteroskedasticity in bond and stock returns. We use a moving block bootstrap, which Maddala

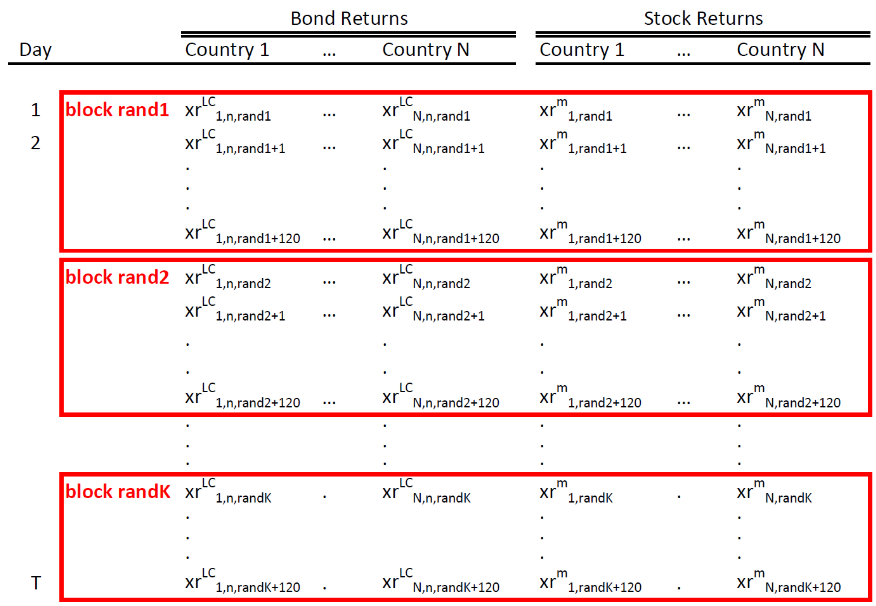
(2001) and Lahiri (1999) argue has superior properties to account for time series correlation. Let N be the number of countries and T be the length of the time series. We define $T - 120$ overlapping blocks of length 120 days, where we use the same blocks for both bond returns $xr_{i,n,t}^{LC}$ and stock returns $xr_{i,t}^{m}$ and we use the same blocks for all countries. Figure A3, Panel (A) illustrates how we define overlapping blocks on the actual data.

Figure A3: Bootstrap Bond and Stock Returns

(A) Defining Overlapping Blocks on Original Data



(B) Defining Bootstrap Bond and Stock Returns



Note: This figure illustrates the moving block bootstrap to generate bootstrapped LC bond and local stock log excess returns (Maddala (2001)). $xr_{i,n,t}^{LC}$ denotes the log excess return over the 91 calendar day period ending on day t for the country i LC bond with remaining time to maturity n. $xr_{i,t}^{m}$ denotes the log excess return over the 91 calendar day period ending on day t on the local equity benchmark in excess of a 3-month T-bill. $rand1$, $rand2$, ... , $randK$ are iid random variables drawn uniformly from the integers between 1 and $T - 120$. We then define bootstrap returns as the sequence of block $rand1$, followed by block $rand2$,... up to block $randK$.

Define $K = \lfloor T/120 \rfloor$, such that a combination of K blocks will generate a bootstrap sample of length $K \times 120 \approx T$. Because T is not generally a multiple of 120, we round K down to be conservative. We then generate bootstrap samples $xr_{i,n,t}^{LC,boot}$ and $xr_{i,t}^{m,boot}$ by randomly drawing K blocks and concatenating them. Formally, we draw iid random variables $rand1$, $rand2$, ... , *randK* uniformly from the integers between 1 and $T - 120$ and define the bootstrap returns as the sequence of blocks $rand1$, $rand2, ..., randK$. Figure A3, Panel (B) illustrates the construction of the bootstrapped bond and stock returns. Because we use the same blocks across all countries, the bootstrap sample preserves the correlation of bond and stock returns across countries. We choose a block length of 120 trading days as a trade-off between capturing the serial correlation of overlapping returns (which are defined using 91 calendar days) and having a sufficient number of blocks to generate plausible variation across the bootstrapped samples.

Having generated a bootstrap sample, we follow the same estimation procedure as in the actual data. We re-estimate Eqn. (3) country-by-country on the bootstrapped data:

$$xr_{i,n,t}^{LC,boot} = a_i + \beta^{boot}(bond_i, stock_i) \times xr_{i,t}^{m,boot} + \epsilon_{i,t}^{boot}. \tag{A1}$$

Figure A2 and Table AII report the standard deviation of $\hat{\beta}^{boot}(bond_i, stock_i)$ across 500 independent bootstrap samples.

A.4 Robustness Checks for the Main Empirical Results

A.4.1 Monte-Carlo Standard Errors

We now show that our benchmark regression in Table II column (1) remains robustly for a range of different distributional assumptions for the residuals. Because we only have 28 countries, asymptotic standard errors are likely to be inappropriate in our setting. We therefore explore a range of different distributional assumptions and how they affect standard errors in Monte Carlo simulations. We explore the implications of different assumptions for the distribution of residuals in a series of Monte Carlo simulations, including normal and wild Bootstrap distributions. The results from that analysis are collected in Appendix Table AIII. This table starts by showing asymptotic standard errors, with and without a Huber-White heteroskedasticity adjustment, in rows (1) and (2) for comparison. Next, row (3) reports the standard error from a simple Monte Carlo simulation that assumes that residuals are normally distributed with residuals for EM and DM countries drawn from the same distribution. Rows (4) through (9) gradually relax these restrictive assumptions on the distribution of residuals, allowing EM and DM residuals to be drawn from separate distributions and allowing for non-zero correlations between EM and DM residuals. Row (4) allows for separate standard deviations for the EM and DM residuals. Rows (5) through (9) drop the assumption that residuals are uncorrelated across all country pairs. In rows (5) through (9), Monte Carlo EM residuals are drawn from a multivariate normal with correlation listed in the fourth column. Monte Carlo DM residuals are similarly drawn from a multivariate normal with correlation listed in the fourth column. The EM and DM distributions are independent of each other. Rows (10) replaces the assumption of normally distributed residuals and instead assumes that residuals are drawn from the empirical distribution. Rows (11) and (12) report standard errors for variants of the bootstrap distribution, assuming that residuals for EM and DM countries are drawn from separate distributions (row (11)) and using a wild bootstrap (row (12)). Even though the standard errors vary across the various specifications, our main finding again remains highly statistically significant.

The details for the wild bootstrap procedure in row (11) are as follows. We generate bootstrapped LC bond-stock betas according to a wild bootstrap that accounts for heteroskedasticity (Davison and Hinkley (1997)). Let b_0 and b_1 denote the point estimates from regressing LC bond-stock betas onto LC debt shares in actual data:

$$\beta\left(bond_i, stock_i\right) \quad = \quad b_0 + b_1 s_i^{TOT} + \varepsilon_i, \tag{A2}$$

that is b_0 and b_1 are the estimated constant and coefficient shown in Table II, column (1). We use ε_i to denote the residual for country i estimated on actual data. The bootstrapped LC bond-stock beta $\beta\left(bond_i, stock_i\right)^{boot}$ is then defined as:

$$\beta\left(bond_i, stock_i\right)^{boot} \quad = \quad b_0 + b_1 s_i^{TOT} + X_i \varepsilon_i, \tag{A3}$$

where X_1, X_2,..., X_N are random variables that we draw independently from a standard normal distribution with with mean zero and variance one. The conditional mean of $\beta\left(bond_i, stock_i\right)^{boot}$ is therefore $b_0 + b_1 \beta s_i^{TOT}$ as in a standard parametric bootstrap and the conditional variance of the bootstrap residual is $\mathbb{V}\left(X_i \varepsilon_i\right) = \varepsilon_i^2$. The wild bootstrap preserves the volatility of the residual and flexibly addresses heteroskedasticity in residuals, similar to situations when Huber-White heteroskedasticity-robust standard errors would be used.

Table AIII: Monte Carlo Standard Errors

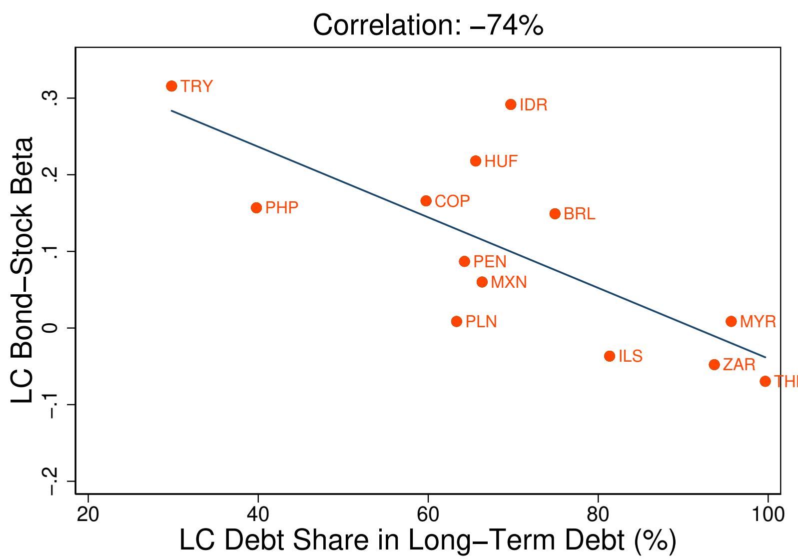
	Method	Distribution	Separate Distributions for EM/DM	Correlation	Standard Error	T-statistic	95th Percentile
(1)	Asymptotic OLS				0.085	-3.377	-0.120
(2)	Asymptotic Huber-White				0.103	-3.377	-0.120
(3)	Monte Carlo	Normal	no	0	0.083	-3.430	-0.150
(4)	Monte Carlo	Normal	yes	0	0.086	-3.334	-0.144
(5)	Monte Carlo	Normal	yes	0.1	0.087	-3.296	-0.145
(6)	Monte Carlo	Normal	yes	0.2	0.091	-3.157	-0.136
(7)	Monte Carlo	Normal	yes	0.3	0.099	-2.899	-0.120
(8)	Monte Carlo	Normal	yes	0.4	0.109	-2.619	-0.107
(9)	Monte Carlo	Normal	yes	0.5	0.124	-2.307	-0.081
(10)	Monte Carlo	Bootstrap	no	0	0.046	-6.256	-0.212
(11)	Monte Carlo	Bootstrap	yes	0	0.081	-3.532	-0.247
(12)	Monte Carlo	Wild Bootstrap	N/A	0	0.100	-2.860	-0.120

Note: This table uses Monte Carlo simulations to investigate the robustness of the baseline regression $\beta_i = a_i + b_i s_i + e_i$ reported in Table II column (1) in the main paper. Row (1) of this table repeats the non-robust asymptotic standard error from Table II for comparison. Row (2) presents the asymptotic Huber-White heteroskedasticity-robust standard error. For rows (3) through (12) we report the standard deviation across 10000 independent Monte Carlo simulations, the ratio of the point estimate reported in Table II divided by the Monte Carlo standard deviation (t-statistic), and the 95th percentile for the slope coefficient across the same 10000 simulations. Row (3) draws Monte Carlo residuals from a normal distribution with mean zero and standard deviation matching that of the empirically estimated residuals. The variance of empirical residuals is estimated as the sum of squared residuals divided by 26 to adjust for the fact that two parameters are estimated. Row (3) uses the same standard deviation of residuals for EMs and DMs. Residuals are drawn independently across countries, so the correlation of Monte Carlo residuals is zero for any country pair. Row (4) modifies the assumption that residuals for EMs and DMs have the same standard deviation, and instead draws Monte Carlo residuals with separate standard deviations for EMs and DMs. The variance of residuals for EMs is estimated as the sum of squared residuals for EMs divided by (17-2), where we make a degrees of freedom adjustment for two estimated parameters. The variance of residuals for DMS is similarly estimated as the sum of squared residuals for DMs divided by (11-2). Rows (5) through (9) drop the assumption that residuals are uncorrelated across all country pairs. In rows (5) through (9), Monte Carlo EM residuals are drawn from a multivariate normal with correlation listed in the fourth column. Monte Carlo DM residuals are similarly drawn from a multivariate normal with correlation listed in the fourth column. The EM and DM distributions are independent of each other. Row (10) generates Monte Carlo draws of β_i using a bootstrap procedure. The residuals are resampled independently and with replacement from the empirical distribution of residuals. Row (11) uses a similar bootstrap procedure but allows for different residual distributions for EMs and DMs. The EM residuals are resampled independently and with replacement from the empirical distribution of EM residuals. The DM residuals are resampled independently and with replacement from the empirical distribution of DM residuals. Row (12) uses a wild bootstrap procedure to flexibly account for heteroskedasticity in residuals. For every country i, the resampled residual equals a standard normal with mean zero and standard deviation scaled by the empirical estimated residual for country i. Bootstrap residuals are drawn independently across all country pairs.

A.4.2 Long-Term Debt

The cross-sectional relationship between LC bond-stock betas and LC debt shares is robust to measuring the LC debt share only in long-term debt, as shown in Figure A4. We obtain face values and issuance dates for all historical individual sovereign bond issuances from *Bloomberg* for 14 emerging markets and estimate the long-term LC debt share as the outstanding amount of LC debt with five or more years remaining to maturity relative to all outstanding debt with five or more years remaining to maturity.

Figure A4: LC Debt Share in Long-Term Debt versus Bond-Stock Beta

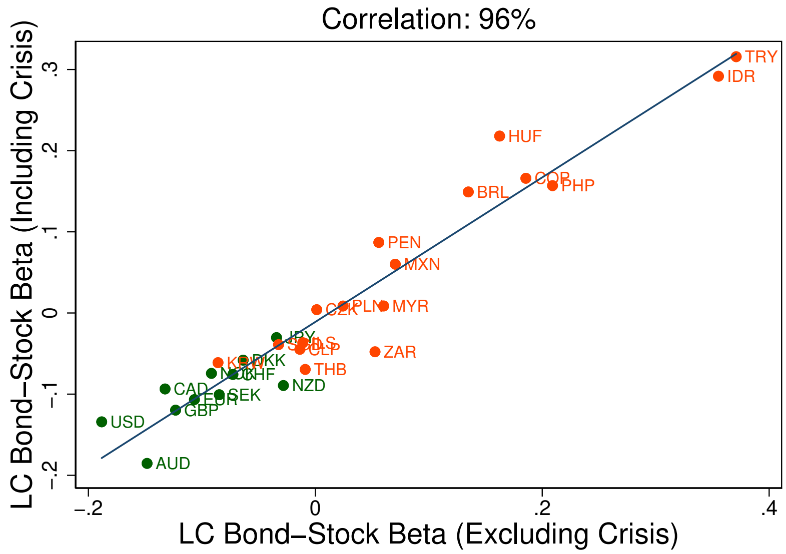


Note: This figure plots the LC bond-stock beta on the y-axis and the share of LC debt in all outstanding long-term debt on the x-axis. Long-term debt is defined as having a remaining time to maturity of five or more years. The share of LC debt in long-term debt is estimated from individual bond issuance data from *Bloomberg*.

A.4.3 Excluding the Financial Crisis

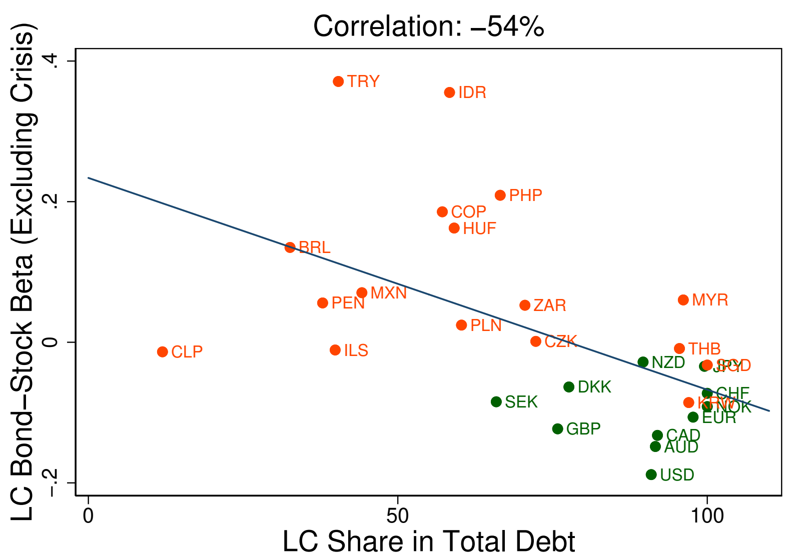
One important period in the middle of our sample is the financial crisis of 2008—2009. While this period marked an important recession for the US and many other countries, we show in this section that our main empirical results are not driven by the financial crisis. Figure A5 shows our baseline LC bond-stock beta on the y-axis against a LC bond-stock beta excluding the financial crisis period on the x-axis. We see that the bond-stock betas are extremely similar when excluding the financial crisis, indicating that our key bond cyclicality measure is not driven by a small number of observations. Figure A6 shows that our main stylized fact in Figure 1 remains unchanged if we exclude the crisis period in our construction of LC bond betas.

Figure A5: Local Currency Bond Betas Excluding 2008—2009



Note: This figure shows LC bond-stock betas excluding the period 2008–2009 on the x-axis and LC bond-stock betas for the full sample (including 2008—2009) on the y-axis.

Figure A6: Local Currency Debt Shares and Bond Betas Excluding 2008—2009



Note: This figure differs from Figure 1 only in that it excludes 2008—2009 from the computation of LC bond betas on the y-axis.

A.4.4 Adjusting for FX hedging errors

In Section A, we calculated the LC bond excess return over the local T-bill rate in local currency units. We discussed that from the dollar investor's perspective, these excess returns approximately hedge the LC fluctuation against the US dollar for the holding period between quarter t and $t+1$. In this section, we re-calculate the bond-stock beta after adjusting for these FX hedging errors for the USD investor.

In particular, suppose that the USD investor invests \$1 in the LC bond at t and funds the position by shorting \$1 of the LC T-bill. At $t+1$, the gross USD return on the LC bond is

$$\frac{P^{LC}_{i,n-1,t+1}}{P^{LC}_{i,n,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \exp[\tau_{i,n,t}y^{LC}_{i,n,t} - (\tau_{i,n,t}-1)y^{LC}_{i,n-1,t+1}]\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}},$$

where $P^{LC}_{i,n,t}$ denotes the price of the n-quarter LC bond at time t in country i, and $\mathcal{E}_{i,t}$ denotes the LC exchange rate defined as USD per LC units, so an increase in $\mathcal{E}_{i,t}$ corresponds to a LC appreciation against the USD. Recall that $\tau_{i,n,t}$ is equal to 5 years. The USD cost of shorting the LC T-bill from time t to time $t+1$ is:

$$\frac{1}{P^{LC}_{i,1,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \exp(-y^{LC}_{i,1,t}/4)\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}.$$

So the exact USD excess return of going long the LC bond and shorting the LC T-bill becomes:

$$\tilde{xr}^{LC}_{i,n,t+1} = \frac{P^{LC}_{i,n-1,t+1}}{P^{LC}_{i,n,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} - \frac{1}{P^{LC}_{i,1,t}}\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}\left[\exp[\tau_{i,n,t}y^{LC}_{i,n,t} - (\tau_{i,n,t}-1)y^{LC}_{i,n-1,t+1}] - \exp(-y^{LC}_{i,1,t}/4)\right].$$

Similarly, for a USD investor, the USD excess return of going long in the LC equity and shorting the LC T-bill is:

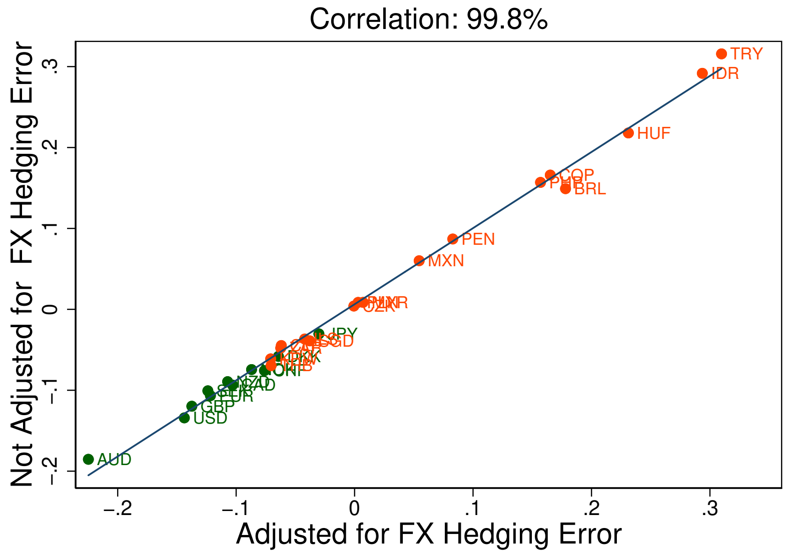
$$\tilde{xr}^{m}_{i,t+1} = \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}[P^m_{t+1}/P^m_t - \exp(-y^{LC}_{i,1,t}/4)].$$

We estimate the bond-stock betas adjusted for FX hedging errors by running the regression:

$$\tilde{xr}^{LC}_{i,n,t} = a_i + \tilde{\beta}(bond_i, stock_i) \times \tilde{xr}^{m}_{i,t} + \epsilon_{i,t}.$$

Figure A7 shows that adjusting these FX hedging errors has no effect on the estimated bond-stock betas. The correlation between the bond-stock beta in local currency units (y-axis) and the bond-stock beta after adjusting for the FX hedging errors (x-axis) is 99.8%.

Figure A7: LC Bond-Stock Beta Adjusting for FX Hedging Errors



Note: On the horizontal axis, we plot the LC bond-stock beta using the bond and stock dollar excess returns after adjusting FX hedging errors, as described in Section A.4.4. On the vertical axis, we plot our baseline LC bond-stock beta in local currency units without adjusting for FX hedging errors.

A.4.5 Larger sample using the inflation-GDP beta

Our main sample in the paper is constrained by the availability of long-term LC bond yields to estimate bond-stock betas. We can extend our sample to over 100 countries by measuring the realized inflation-GDP beta. To obtain standardized data across as many countries as possible, we use the inflation and GDP data from the World Bank World Development Indicator (WDI), which are available at the annual frequency. In order to obtain more precise estimates, we use a longer sample from 1980 to 2017. We require at least 20 observations for a country to be included in the sample, which leaves us with 107 sample countries.

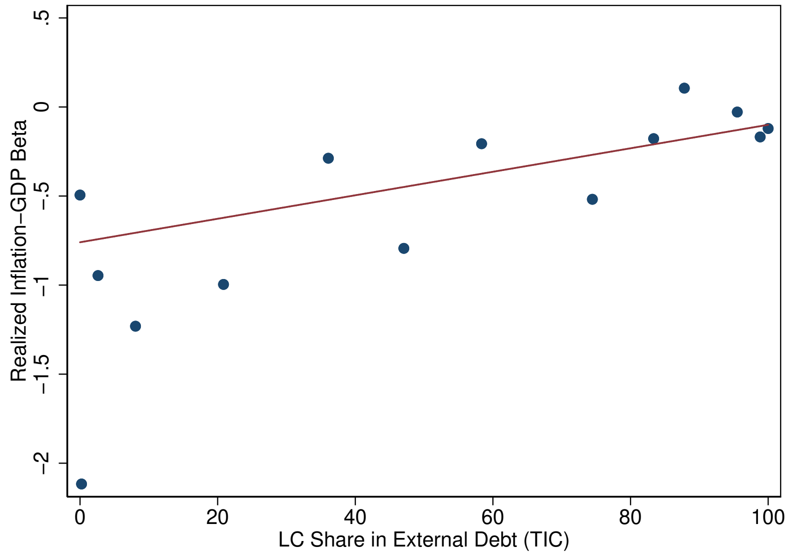
Similar to the realized inflation beta with respect to industrial production as estimated by Eqn. (5), we can estimate the realized inflation beta with respect to GDP by running the following regression.

$$\Delta\pi_{i,t} = a_i + \beta(\pi_i, GDP_i)\Delta GDP_{i,t} + \epsilon_t, \qquad \text{(A4)}$$

where $\Delta\pi_{i,t}$ is the yearly change in the year-over-year inflation rate, and $\Delta GDP_{i,t}$ is the annual change in the GDP growth rate. The coefficient $\beta(\pi_i, GDP_i)$ measures the realized inflation cyclicality with respect to GDP for country i. Before estimating the inflation-output relation for each country, we winsorize the top and bottom 1 % of the inflation rate and the GDP growth rate across countries to remove extreme outliers. Having estimated inflation-output betas for each country, we do not do any further winsorization.

Figure A8 is a binscatter plot showing a positive relationship between the the realized inflation-GDP beta and LC debt share in external debt based on the *TIC* data. Regression results are reported in Table AIV. We can see that the coefficient on the realized inflation-GDP is positive and significant. These findings support our main analysis in Table 2, because if inflation drives down the value of LC bonds we would expect realized inflation-output betas to be inversely related with LC bond-stock betas.

Figure A8: Binscatter Plot of Realized Inflation-GDP Beta vs. LC Debt Share



Note: On the vertical axis, we plot the realized inflation-GDP beta. On the horizontal axis, we plot the share of LC debt in total external debt in %, measured by *TIC*. The binscatter is plotted with 20 bins.

Table AIV: Realized Inflation-GDP Betas and LC Debt Shares in External Debt

	(1)	(2)
Realized Inflation-GDP Beta	$\beta(\pi_i, GDP_i)$	$\beta(\pi_i, GDP_i)$
s^{TIC}	0.66***	0.60***
	(0.19)	(0.22)
log(GDP)		0.01
		(0.06)
FX Regime		-0.00
		(0.00)
Commodity Share		-0.33***
		(0.10)
Constant	-0.76***	0.01
	(0.14)	(0.55)
Observations	107	107
R-squared	0.08	0.20

Note: This table shows the regression results of the realized inflation-GDP beta on the LC debt share in external debt based on *TIC*. Column (1) shows the univariate specification without controls. Column (2) shows the specification with controls. Standard errors used in all regressions are the OLS standard errors. Significance levels indicated by *** p<0.01, ** p<0.05, * p<0.1.

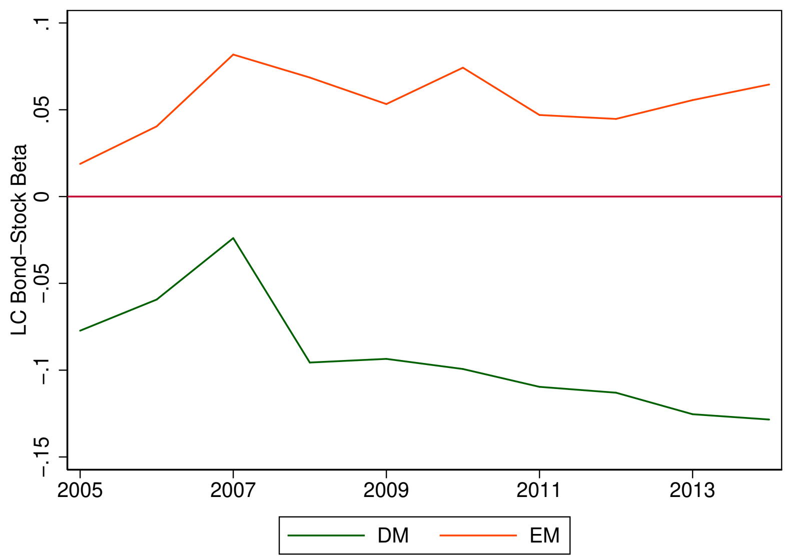
A.4.6 Time-varying betas and LC debt shares

We next show that our results are stable across different time periods. To start, we show that individual countries' bond-stock betas are stable over time. We estimate time-varying LC bond-stock betas, $\beta_t(bond_i, stock_i)$, using five-year rolling windows between $t-5$ and t. Panel (A) of Figure A9 shows the average bond-stock beta for developed and emerging markets. The average beta for developed countries fluctuated between -0.15 and 0, and the average beta for emerging market fluctuated between 0 and 0.1. Panel (B) of Figure A9 plots the cross-country rankings of the bond-stock betas between 2008 and 2014. We can see that the cross-sectional ranking is very persistent. The average pairwise rank correlation between 2008 and 2014 is 92%.

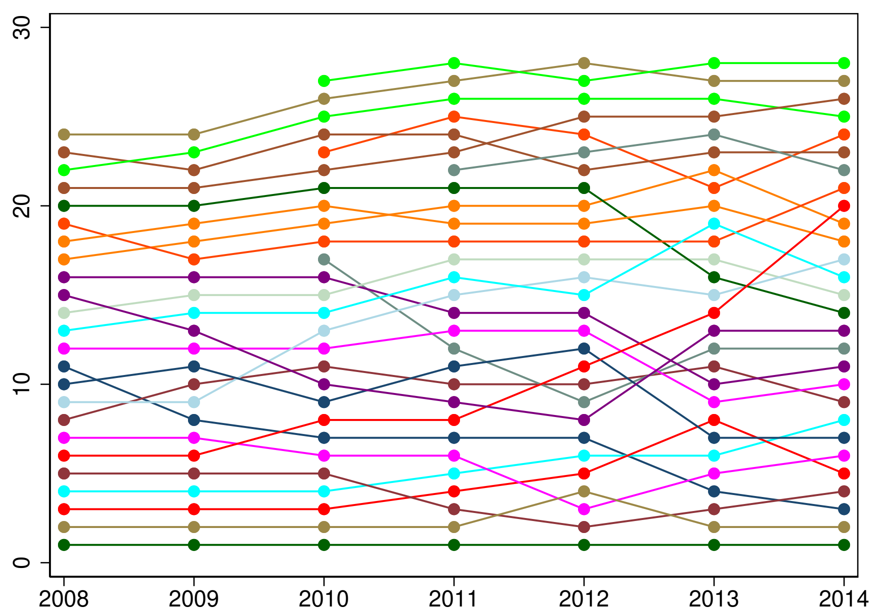
We next run the cross-sectional regressions of $\beta_t(bond_i, stock_i)$ on the LC debt share at time t for every year in our sample. The regression results are shown in Table AV. The coefficient on the LC debt share is negative and statistically significant for all sample years.

Figure A9: Time variations in the bond-stock beta

(A) Rolling LC Bond-Stock Betas



(B) Ranking of Rolling LC Bond-Stock Betas



Note: Panel (A) plots the average rolling LC bond-stock beta over time for developed markets (DM) and emerging markets (EM). The LC bond-stock beta at time t is calculated using a five-year rolling window between $t-5$ and t. Panel (B) plots the cross-country ranking of the five-year rolling LC bond-stock betas over time, with each color indicating a sample country.

Table AV: Regression of the LC Bond-Stock Beta on the LC Debt Share by Year

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
s^{TOT}	-0.511***	-0.488***	-0.379***	-0.399***	-0.375***	-0.332***	-0.266***	-0.247**	-0.260**	-0.240**
	(0.0915)	(0.101)	(0.0904)	(0.108)	(0.106)	(0.0985)	(0.0851)	(0.0920)	(0.106)	(0.111)
Constant	0.391***	0.391***	0.339***	0.308***	0.283***	0.249***	0.180***	0.165**	0.176**	0.164*
	(0.0778)	(0.0860)	(0.0758)	(0.0881)	(0.0865)	(0.0760)	(0.0644)	(0.0699)	(0.0805)	(0.0836)
Observations	20	20	22	24	24	26	27	27	27	27
R-squared	0.634	0.564	0.468	0.381	0.362	0.321	0.281	0.225	0.194	0.157

Note: This table shows the yearly regression results of the LC bond-stock beta estimated using a five-year rolling window between $t-5$ and t on the LC debt share in year t.

A.5 LC Bond Return Comovement with US Stock Returns

We now show empirically that the LC bonds with the best hedging value for the domestic government are risky for international lenders. In this analysis, we proxy for domestic agents' marginal utility of consumption with the local log excess stock return and for international lenders' SDF with the US log excess stock return. We decompose the local log excess stock return into a global and an idiosyncratic component according to:

$$xr_{i,t}^{m} = a_i + \beta(stock_i, stock_{US}) \times xr_{US,t}^{m} + xr_{i,t}^{idio}. \tag{A5}$$

We define the systematic global component of local stock returns as the fitted value of Eqn. (A5):

$$xr_{i,t}^{G} = \beta(stock_i, stock_{US}) \times xr_{US,t}^{m}.$$

It is conceivable that LC bond returns co-move with domestic stock returns only through the idiosyncratic component, $xr_{i,t}^{idio}$, that is orthogonal to US stock returns. In this case, LC bonds would have zero covariance with US stock returns and present no systematic risk to international lenders, and our main channel would not be operative.

To alleviate this concern, we show in two ways that the LC bonds with the best hedging benefit for the domestic borrower are indeed risky for international lenders. First, we directly estimate the beta of LC bond returns with respect to US stock returns from a regression:

$$xr_{i,n,t}^{LC} = a_i + \beta(bond_i, stock_{US}) \times xr_{US,t}^{m} + \epsilon_{i,t}. \tag{A6}$$

Panel (A) of Figure A10 shows that $\beta(bond_i, stock_{US})$ is highly correlated with our baseline measure of bonds' hedging value for the domestic borrower, $\beta(bond_i, stock_i)$, estimated in Eqn. (3). The cross-country correlation of these two different bond betas equals 89%, clearly supporting a link between the domestic borrower's hedging value and international lenders' risk of holding LC bonds.

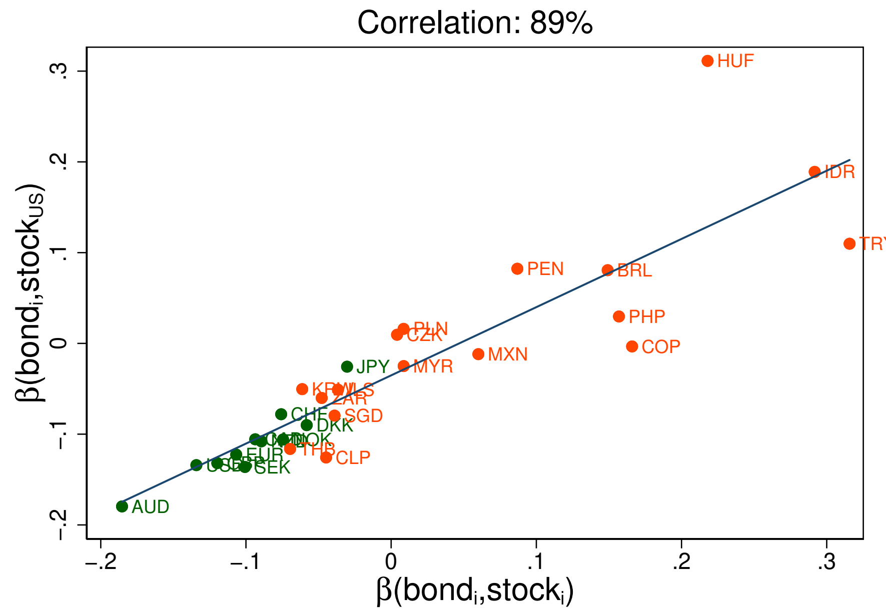
Second, we estimate LC bond excess return loadings on the systematic global component of domestic stock returns using the regression:

$$xr_{i,n,t}^{LC} = a_i + \beta(bond_i, stock_{i,US}^{G}) \times xr_{i,t}^{G} + \epsilon_{i,t}. \tag{A7}$$

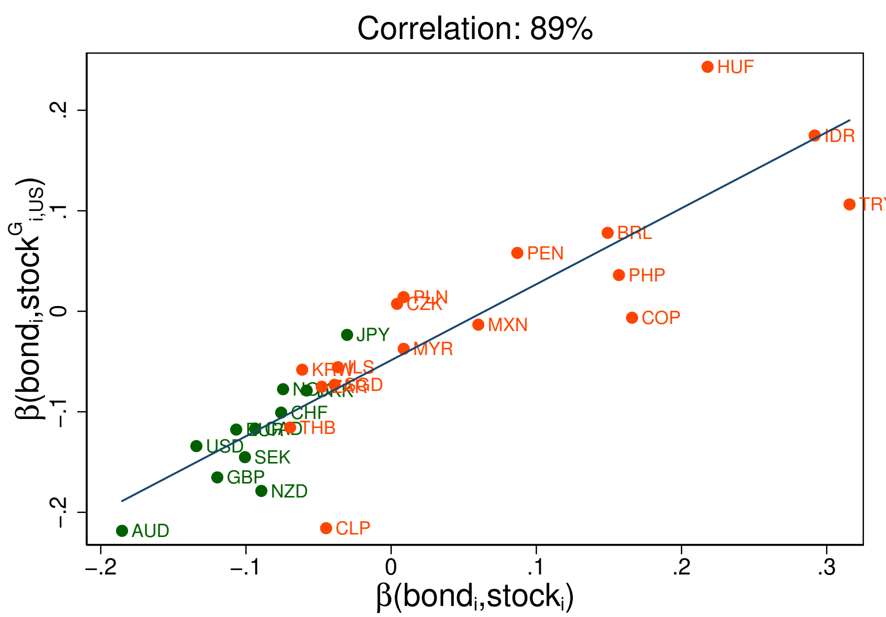
Panel (B) of Figure A10 shows that $\beta(bond_i, stock_{i,US}^{G})$ is 89% correlated with our baseline measure of bonds' hedging value for the domestic borrower $\beta(bond_i, stock_i)$. In other words, LC bond returns co-move with the global component of local LC stock returns.

Figure A10: Local and Global Risks of LC Bonds

(A) Beta onto US Stock Returns



(B) Beta onto Global Component of Local Stock Returns



Note: Panel (A) plots on the y-axis the regression beta of LC bond excess returns on US S&P stock excess returns, $\beta(bond_i, stock_{US})$, estimated from Eqn. (A6). Panel (B) plots on the y-axis the regression beta of LC bond excess returns on the global component of local LC bond returns, $\beta(bond_i, stock^G_{i,US})$, estimated from Eqn. (A7). Our baseline one-factor LC bond-stock beta with respect to the local stock market, estimated from Eqn. (3), is shown on the x-axis in both panels. The bivariate correlation across countries is shown in the figure title.

A.6 Testing the CAPM

A.6.1 GRS Test of the CAPM

The paper treats stock market betas as proxies for expected excess returns. We now estimate a standard Gibbons et al. (1989) (GRS) test for the CAPM, with the US stock market as a proxy for total wealth.

We start by sorting our countries into five equal-sized portfolios, sorted by their LC bond betas with respect to the US stock market. We obtain quarterly bond excess returns (not overlapping) for these five portfolios. Due to our short sample period, it is unsurprising that average excess returns are noisy.

We test the CAPM with the GRS statistic, which Campbell (2017) shows can be written as:

$$GRS = \frac{T-N-1}{N}\frac{(Sharpe^{LC,tangency})^2 - (Sharpe^{m}_{US})^2}{1+(Sharpe^{m}_{US})^2}. \quad \text{(A8)}$$

Here, $Sharpe^{LC,tangency}$ is the Sharpe ratio of the tangency portfolio of the LC bond portfolios, $Sharpe^{m}_{US}$ is the Sharpe ratio of the US equity market, $T = 42$ is the number of quarterly returns, and $N = 5$ is the number of portfolios. The GRS statistic, hence, increases in the distance between the Sharpe ratios for the tangency portfolio and the US equity market.

We estimate the tangency portfolio Sharpe ratio from the portfolio returns as in Campbell (2017) Chapter 2.2.3. This gives a tangency Sharpe ratio of $Sharpe^{LC,tangency} = 0.52$, compared to a US equity market Sharpe ratio of $Sharpe^{m}_{US} = 0.17$, over our sample period 2004 to 2015. The Sharpe ratio for the LC bond tangency portfolio hence exceeds the equity Sharpe ratio over our short sample period. However, the tangency Sharpe ratio is very close to the US equity Sharpe ratio of 0.56 reported in Campbell (2003) for a longer sample that is conventionally used to obtain a more precise estimate of average US equity excess returns. The proximity between the tangency Sharpe ratio and the US equity Sharpe ratio from this longer sample is an intuitive indication that the difference between tangency and US equity Sharpe ratios over the shorter sample is not statistically significantly different.

Substituting the values for $Sharpe^{LC,tangency}$, $Sharpe^{m}_{US}$, T, and N into (A8) gives a value for the GRS statistic of $GRS = 1.72$. Comparing this value to the critical values of a $F_{N,T-N-1}$ distribution gives a p-value of 0.16, showing formally that we cannot reject CAPM at any conventional significance level.

A.6.2 GMM Risk Premium Estimation

We next make use of the fact that our assets of interest are bonds and that we can use quoted bond yields to construct ex ante measures of LC bond risk premia. Ex ante bond risk premia may be more precisely measured than the ex post average returns over a limited sample used for the GRS test. We find that ex ante LC bond risk premia have a statistically and quantitatively significant relationship with US stock market betas across countries. This estimation is similar to the GRS test in Section A.6.1, because we seek to estimate whether investors require a higher risk premium for LC bonds that comove more with the US stock market. Further, we want to understand whether this price of risk is statistically distinguishable from the average US equity risk premium.

A concrete example makes clear the advantage of ex ante risk premia measures based on bond yields, whereas realized bond returns are noisy measures of ex ante expected risk premia over our short sample. For instance, the US had extremely low government bond yields throughout our sample, indicating that investors required low risk premia for holding US Treasuries. However, US Treasury yields dropped even lower during our sample and, in particular, during the financial crisis, an event that would have been very hard to predict ex ante. As a result, looking at US excess returns, it would appear as if the US had a high risk premium, whereas clearly markets price a very low risk premium into US Treasuries.

We estimate a regression of ex ante average expected risk premia onto the beta of LC bond returns with respect to the US stock market, while accounting for the fact that the betas on the right-hand side of this regression are not known but instead must be estimated.

For comparison and to set the stage, we first estimate this relationship in two steps without accounting for generated regressors. As a first-step, we estimate country-by-country regressions:

$$xr_{i,n,t}^{LC} = \alpha_i + \beta_i xr_{US,t}^m + \epsilon_{i,t}, \tag{A9}$$

using daily data on overlapping 1-quarter holding returns. Because we use daily overlapping returns, the average number of return observations per country is high at 2513. For comparison, the maximum number of return observations is 2608, so our data is close to a balanced panel. Let $\overline{RP}_{i,n}$ denote the average ex ante risk premium estimated for country i. In a second step, we then estimate the regression:

$$\overline{RP}_{i,n} = \mu + \kappa\beta_i + u_i. \tag{A10}$$

The coefficient, κ, estimates the cost of exposure to the US stock market and is the coefficient of interest.

To estimate α_i, β_i, μ, and κ in a single step while accounting for estimation error in the first stage, we define the following Generalized Method of Moments (GMM) moments, which we expect to have a population mean of zero:

$$g_{i,t} = \begin{cases} \overline{RP}_{i,n} - \mu - \kappa\beta_i & \text{for} \quad 1 \leq i \leq N \\ \left(\overline{RP}_{i,n} - \mu - \kappa\beta_i\right)\beta_i & \text{for} \quad N+1 \leq i \leq 2N \\ xr_t^{LC} - \alpha_i - \beta_i xr_{US,t}^m & \text{for} \quad 2N+1 \leq i \leq 3N \\ \left(xr_{i,t}^{LC} - \alpha_i - \beta_i xr_{US,t}^m\right) xr_{US,t}^m & \text{for} \quad 3N+1 \leq i \leq 4N \end{cases} \tag{A11}$$

Here, N denotes the number of countries in the sample and the parameter vector to be estimated is:

$$\begin{aligned} b &= [\mu, \kappa, \boldsymbol{\alpha}, \boldsymbol{\beta}]', \\ \boldsymbol{\alpha} &= [\alpha_1, \alpha_2, ..., \alpha_N], \\ \boldsymbol{\beta} &= [\beta_1, \beta_2, ..., \beta_N]. \end{aligned}$$

The first $2N$ moment conditions in (A11) are for the cross-sectional regression in the second stage. Moment conditions $2N+1$ through $4N$ are for the first-stage regressions. In sample, the $4N$ moments (A11) cannot all simultaneously be set to zero, because we only have $2N+2$ parameters. The GMM estimator $\hat{b}$ is defined by setting:

$$A \times \frac{1}{T}\sum_{t=1}^{T} g_t\left(\hat{b}\right) = 0, \tag{A12}$$

where A is a weighting matrix of size $(2N+2) \times 4N$ that has full rank. It is a standard result for

GMM that the estimated parameter vector $\hat{b}$ has asymptotic distribution

$$\hat{b} \sim \mathcal{N}(b_0, V) \tag{A13}$$
$$V = T^{-1}(AD)^{-1} ASA' (AD)^{-1'}, \tag{A14}$$

where b_0 is the true underlying parameter value, $D = E\left[\frac{\partial g}{\partial b'}\right]$ is the sample average of the derivative of g with respect to the parameter vector, b, and S is the spectral density matrix of g_t at frequency zero.

We implement GMM with weighting matrix $A = [(2N+2) \times 4N]$ that ensures that the GMM estimates for μ and κ agree with the point estimates from the two-step procedure. This requirement pins down the weighting matrix:

$$A = \begin{bmatrix} 1_{1\times N} & 0_{1\times N} & 0_{1\times 2N} \\ 0_{1\times N} & 1_{1\times N} & 0_{1\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & \mathcal{I}_{2N} \end{bmatrix}. \tag{A15}$$

Here $0_{M\times P}$ and $1_{M\times P}$ define block matrices of all zeros and ones with size $[M \times P]$, respectively. We use $\mathcal{I}_{2N}$ to denote the identity matrix of size $2N$. For our application, we use the consistent estimator for D:

$$\hat{D} = \begin{bmatrix} -1_{N\times 1} & -\boldsymbol{\beta} & 0_{N\times N} & -\kappa I_N \\ -\boldsymbol{\beta} & -\boldsymbol{\beta}^2 & 0_{N\times N} & -2\kappa \times diag(\boldsymbol{\beta}) \\ 0_{N\times 1} & 0_{N\times 1} & -\mathcal{I}_N & -\mathcal{I}_N \sum_{t=1}^{T} xr_{US,t}^{m} T^{-1} \\ 0_{N\times 1} & 0_{N\times 1} & -\mathcal{I}_N \sum_{t=1}^{T} xr_{US,t}^{m} T^{-1} & -\mathcal{I}_N \sum_{t=1}^{T} \left(xr_{US,t}^{m}\right)^2 T^{-1} \end{bmatrix}, \tag{A16}$$

where $diag(\boldsymbol{\beta})$ denotes the matrix with the elements of $\boldsymbol{\beta}$ along the diagonal. We estimate the upper left $[2N \times 2N]$ submatrix of S from the cross-section of countries, with the assumption that (β_i, u_i) are independent but not necessarily identically distributed. We also assume that $g_{i,t}, 1 \le i \le 2N$ are independent of $g_{j,t}, 2N < j < 4N$, so we can set the upper-right $2N \times 2N$ and the lower-left $2N \times 2N$ block matrices of the spectral density matrix, S, to zero. We cannot estimate the upper-right $2N \times 2N$ and the lower-left $2N \times 2N$ block matrices of the spectral density matrix, S, because $\overline{RP}_{i,n}$ is constant over time for each country. The spectral density for moments $2N+1$ through $4N$ is estimated from the time series with a Newey-West kernel with m lags to account for serial correlation and overlapping return observations:

$$\hat{S} = \begin{bmatrix} \mathcal{I}_N \hat{s}_1 & \mathcal{I}_N \hat{s}_{12} & 0_{N\times 2N} \\ \mathcal{I}_N \hat{s}_{12} & \mathcal{I}_N \hat{s}_2 & 0_{N\times 2N} \\ 0_{2N\times N} & 0_{2N\times N} & T^{-1}\sum_{t=1}^{T}\left(\tilde{g}_t\tilde{g}_t' + \sum_{i=1}^{m}\left(1-\frac{i}{m+1}\right)\left[\tilde{g}_t\tilde{g}_{t-i}' + \tilde{g}_{t+i}\tilde{g}_t'\right]\right) \end{bmatrix}. \tag{A17}$$

Here,

$$\hat{s}_1 = \frac{1}{N-2}\sum_{t=1}^{T}\sum_{i=1}^{N} g_{i,t}^2, \tag{A18}$$

$$\hat{s}_2 = \frac{1}{N-2}\sum_{t=1}^{T}\sum_{i=N+1}^{2N} g_{i,t}^2, \tag{A19}$$

$$\hat{s}_{12} = \frac{1}{N-2}\sum_{t=1}^{T}\sum_{i=1}^{N} g_{i,t}g_{i+N,t}, \tag{A20}$$

and $\tilde{g}_t$ refers to the vector containing elements $g_{2N+1,t}$ through $g_{4N,t}$. We choose a lag length of $m = 120$ days to account for the length of overlapping observations of approximately 60 trading days. A lag length of $m = 120$ days is sufficiently small relative to our overall sample length of 2608 trading days that standard asymptotic standard errors apply.

We then compute the GMM standard errors for μ and κ as follows:

$$SE(\hat{\mu}) = \sqrt{V(1,1)}, \tag{A21}$$

$$SE(\hat{\kappa}) = \sqrt{V(2,2)}. \tag{A22}$$

Table AVI column (1) starts by reporting the estimated regression Eqn. (A10) without accounting for generated regressors. It reports non-robust OLS standard errors. We note that the bond-US stock beta enters with a strongly positive coefficient that is also statistically significant. The results suggest that the price of US stock market risk is 8.96%, that isan asset with a unit beta with respect to the US stock market has a risk premium of 8.96%. This number is very close to and not statistically significantly different from the equity premium of 8.1% reported in Campbell (2003). Column (2) in Table AVI reports results from the GMM procedure, which accounts for generated regressors. The point estimates are identical to column (1) and the standard errors are only slightly larger without affecting statistical significance, as one would expect if the vector of bond betas, $\boldsymbol{\beta}$, is precisely estimated.

Table AVI: GMM: LC Bond Risk Premia onto LC Bond-US Stock Betas

LC Bond Risk Premium	(1) OLS	(2) GMM
β_i	8.96***	8.96***
	(2.54)	(3.27)
Constant	2.81***	2.81***
	(0.29)	(0.38)
Observations	28	28

Note: This table estimates the regression (A10), where LC bond-US stock return betas are estimated via (A9). The specification in column (1) does not account for generated regressors and reports standard OLS standard errors. Column (2) accounts for generated regressors by using the GMM procedure described in Appendix A.6.2. Significance levels are indicated by *** p<0.01, ** p<0.05, * p<0.1.

B Model Appendix

The Model Appendix is structured as follows:

- Appendix B.1 microfounds real exchange rate shocks.
- Appendix B.2 describes how we model stock returns as a factor loading onto output surprises in the calibrated model.
- Appendix B.3 shows that under the assumptions of lump-sum taxes and a representative domestic consumer, inflating away domestically-held LC debt has no effect on domestic real consumption. Inflating away LC debt is only an aggregate transfer of resources to domestic consumers when the debt is owned by international lenders. This insight allows us to focus on externally-held debt throughout the main paper.
- Appendix B.4 derives the first-order conditions.
- Appendix B.5 proves Proposition 1.
- Appendix B.6 describes the numerical solution.
- Appendix B.7 shows that the quantitative results are robust to reasonable variation in parameter values and in particular to allowing separate exchange rate processes for emerging and developed markets.

B.1 Microfounding the Real Exchange Rate

This section describes the goods and preferences microfounding the real exchange rate.

B.1.1 International Consumers

Following Gabaix and Maggiori (2015), we assume that international consumers consume a consumption basket:

$$C_t^* \quad = \quad (A_t^*)^{\mathcal{E}_t} (O_t^*)^{1-\mathcal{E}_t}, \tag{B1}$$

where $\mathcal{E}_t$ is a non-negative, potentially stochastic preference parameter.[35] A_t^* denotes the number of apples and O_t^* the number of oranges consumed by international consumers in periods $t = 1, 2$. We normalize the preference shock in period 1 to one. The period 2 preference shock is log-normally distributed according to Eqns. (25) and (26). To summarize, the distribution of the preference shock is:

$$\begin{aligned} \mathcal{E}_1 &= 1, \\ \mathcal{E}_2 &= exp\left(\varepsilon_2 - \frac{1}{2}\sigma_\varepsilon^2\right), \qquad \text{(B2)} \\ \varepsilon_2 &= \lambda^{\varepsilon,x^*} x_2^* + e_2, \qquad \text{(B3)} \end{aligned}$$

[35] Pavlova and Rigobon (2007) also consider a similar foundation for real exchange rate fluctuations based on preference shocks.

where e_2 is distributed according to:

$$e_2 \quad \sim \quad N(0, \sigma_e^2),$$

independently of x_2 and x_2^*. International consumers' welfare function is given by:

$$U^* \quad = \quad E \sum_{t=1}^{2} (\delta^*)^t \frac{(C_t^*)^{1-\gamma^*}}{1-\gamma^*}. \tag{B4}$$

We assume that the international economy is endowed with an equal amount of apples and oranges in each period. Furthermore, the international economy's endowment of apples and oranges equals $A_1^* = O_1^* = X_1^* = 1$ in period 1 and it equals $A_2^* = O_2^* = X_2^*$ in period 2, where X_2^* follows the distribution described in the main paper. Since the domestic economy is assumed to be small, the effect of domestic bond payoffs on international consumers' consumption is negligible. The international consumers' consumption bundle then equals:

$$\begin{aligned} C_1^* &= A_1^* = O_1^* = 1, \\ C_2^* &= A_2^* = O_2^* = X_2^*. \end{aligned}$$

B.1.2 Domestic Economy

Domestic consumers have preferences over the real domestic consumption bundle and domestic log inflation:

$$U(C_2, \pi_2) \quad = \quad \frac{C_2^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_2^2. \tag{B5}$$

The domestic consumption bundle consists entirely of apples:

$$C_2 \quad = \quad A_2. \tag{B6}$$

The amount of apples consumed in Eqn. (B6) is endogenous, and depends on the exogenous endowment net of real debt repayments, as specified in Eqn. (14) We define the consumption-weighted real exchange rate as the price that international consumers are willing to pay for apples, where the numeraire is one unit of the international consumers' consumption bundle. With (B1), (B4), (B5), and (B6), the real exchange rate equals:

$$\frac{\frac{dU^*}{dA_t^*}}{\frac{dU^*}{dC_t^*}} \quad = \quad \mathcal{E}_t, \tag{B7}$$

showing that the real exchange rate indeed follows the process described in the main paper.

B.2 Bond and Stock Returns

In order to compare bond-stock betas in the model and in the data, we need to model bond and stock returns. Model log excess LC bond return innovations equal the revision to log bond prices from period 1 to period 2 :

$$xr^{LC}_{i,2} - \mathbb{E}xr^{LC}_{i,2} \quad = \quad -\left(\pi_{i,2} - \mathbb{E}\pi_{i,2}\right), \tag{B8}$$

where $i = EM$ or $i = DM$. Model LC bond excess returns are currency hedged, analogously to the empirical analysis in Section I.

We model stocks simply as an asset class whose log dividends are proportional to log domestic output. In order to focus on the role of government bonds as a tool to hedge domestic consumption, we assume that stocks cannot be traded across borders. Specifically, we model log domestic equity return innovations as proportional to log domestic output:

$$xr^{m}_{i,2} - \mathbb{E}xr^{m}_{i,2} \quad = \quad \lambda^{m,x}\left(x_{i,2} - \mathbb{E}x_{i,2}\right). \tag{B9}$$

In our calibration, we set the coefficient $\lambda^{m,x}$ to be consistent with the data. Regressing quarterly local equity excess returns onto log domestic output gives a coefficient of 4, averaged across EMs and DMs, as listed in Table V. The estimated coefficient for EMs is not statistically different from the one for DMs at the 95% level, so we use the average in the calibration for both EMs and DMs.

With Eqns. (B8) and (B9) we obtain a simple relationship between model bond-stock betas and inflation-output betas:

$$\beta^{model}\left(bond_i, stock_i\right) \quad = \quad -\frac{1}{\lambda^{m,x}}\beta^{model}\left(\pi_i, x_i\right), \tag{B10}$$

$$\beta^{model}\left(\pi_i, x_i\right) \quad = \quad \frac{Cov(\pi_{i,2}, x_{i,2})}{\sigma^2_{i,x}}. \tag{B11}$$

The relation in Eqn. (B10) captures the intuition that bond-stock betas have the opposite sign from inflation-output betas and are compressed towards zero, because stocks are more volatile than output.

B.3 Domestic Debt Extension

We now present an extension of the model with domestically-held LC debt. That is, the government can borrow from its own domestic consumers with LC debt in addition to borrowing from international lenders. We show that under the assumptions that the government has access to lump-sum taxes and a representative consumer, inflating away domestically-held LC debt leaves domestic real consumption unchanged. Inflating away LC debt only generates an aggregate transfer of resources to domestic consumers if that debt is held by international lenders. This observation motivates our focus on internationally-held debt throughout the paper.

We assume that the government borrows face value $D^{LC,dom}$ of LC debt from domestic consumers and face value D^{LC} of LC from international lenders at prices $Q^{LC,dom}$ and Q^{LC}. Note that we allow for potentially different bond prices paid by domestic consumers and international lenders. We continue to assume that the government needs to raise external financing $\bar{D}/R^*$ in period 1. To leave period 1 consumption normalized at 1, we assume that proceeds from domestic bond sales are rebated to domestic consumers.

The real amount of domestic goods needed to repay the government debt in period 2 becomes:

$$D_2 \quad = \quad \frac{D^{FC}}{\mathcal{E}_2} + \left(D^{LC} + D^{LC,dom}\right) exp(-\pi_2). \tag{B12}$$

Because the government has access to lump-sum taxes, real period 2 domestic consumption equals the domestic endowment minus real resources needed to repay government debt plus the payoff on the domestically-held LC bond portfolio:

$$C_2 \;=\; X_2 - D_2 + D^{LC,dom} exp(-\pi_2). \tag{B13}$$

Substituting (B12) into (B13) shows that domestic real consumption depends on D^{FC} and D^{LC} but is independent of domestically-held debt $D^{LC,dom}$:

$$C_2 \;=\; X_2 - \left(\frac{D^{FC}}{\mathcal{E}_2} + D^{LC} exp(-\pi_2) \right). \tag{B14}$$

Intuitively, surprise inflation reduces domestic consumers' returns on their LC bond portfolio. However, surprise inflation also reduces the taxes required to repay debt. With lump sum taxes these two effects exactly cancel and domestic consumption is independent of the return on domestically-held debt. The finding that real domestic consumption is independent of domestically-held LC debt makes clear that externally-held debt is the key variable for the equilibrium inflation policy and bond risks.

B.4 First-Order Conditions

Proof of Inflation First-Order Condition with Commitment

We now prove the commitment government's first-order condition characterized by Eqns. (31) and (32). To simplify the derivation, we assume that there is a discrete number of states $j = 1, ..., N$ that are realized with probability f_j. With a discrete number of states f_j takes the role of the probability density function $f(X_2)$ in the main text. We use x_j, π_j etc. to denote the values for log real domestic output and log inflation if state j is realized in period 2. In this section we omit the superscript ^{c} and time period $_2$ subscript and reserve subscripts to indicate the state that has been realized. For simplicity, we first prove Eqn. (31) with the two additional simplifying assumptions that there is only one output shock ($x_j^* = x_j \forall j$) and there is no real exchange rate shock ($\varepsilon_j = 0 \forall j$), before proving the general case. In this simplified special case, the commitment government's problem is to choose the vector π_1,π_2,..., π_N to maximize:

$$\mathbb{E}U = \sum_{j=1}^{N} f_j \left(\frac{C_j^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_j^2 \right), \tag{B15}$$

where consumption in state i is given by:

$$C_i \;=\; \bar{X} exp\left(x_i/\bar{X}\right) - \bar{D} \left(\frac{s}{R^*} \frac{exp(-\pi_i)}{Q^{LC}} + (1-s) \right), \tag{B16}$$

and the LC bond price equals:

$$Q^{LC} \;=\; \sum_{j=1}^{N} f_j M_j^* exp\left(-\pi_j\right). \tag{B17}$$

Here, the international lenders' SDF in state j follows from Eqn. (19) and equals:

$$M_j^* \;=\; \delta^* exp\left(-\gamma^* x_j\right). \tag{B18}$$

The international real risk-free rate satisfies:

$$\frac{1}{R^*} = \sum_{j=1}^{N} f_j M_j^*. \tag{B19}$$

The commitment government chooses the inflation rate in state i such that the marginal benefit of raising inflation in that state equals the marginal cost. The derivative of ex-ante expected utility with respect to log inflation in state i, $\frac{d\mathbb{E}U}{d\pi_i}$, equals:

$$\frac{d\mathbb{E}U}{d\pi_i} = f_i U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \frac{\partial}{\partial Q^{LC}}\left(\sum_{j=1}^{N} f_j \frac{C_j^{1-\gamma}}{1-\gamma}\right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i, \tag{B20}$$

$$= f_i U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \left(\sum_{j=1}^{N} f_j U'(C_j)\frac{\partial C_j}{\partial Q^{LC}}\right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i, \tag{B21}$$

where we use the notation $U'(C_j) = \frac{\partial U}{\partial C_j}(C_j, \pi_j) = C_j^{-\gamma}$. Dividing by the probability f_i and setting $\frac{d\mathbb{E}U}{d\pi_i} = 0$ gives the first-order condition:

$$\alpha\pi_i = U'(C_i)\frac{\partial C_i}{\partial \pi_i} + \frac{1}{f_i}\frac{dQ^{LC}}{d\pi_i}\mathbb{E}\left[U'(C_i)\frac{\partial C_i}{\partial Q^{LC}}\right]. \tag{B22}$$

Differentiaing Eqn. (B17) with respect to π_i shows that:

$$\frac{dQ^{LC}}{d\pi_i} = -f_i M_i^* exp(-\pi_i). \tag{B23}$$

When the domestic output X_2 follows a continuous probability distribution we need to replace f_i by the density $f(X_2)$, π_i by $\pi_2^c(X_2)$, C_i by $C_2^c(X_2)$, and M_i^* by $M_2^*(X_2)$. For brevity, we omit the arguments of π_2^c, C_2^c, and M_2^*, so Eqns. (B22) and (B23) become:[36]

$$\alpha\pi_2^c = U'(C_2)\frac{\partial C_2}{\partial \pi_2^c} + \frac{1}{f(X_2)}\frac{dQ^{LC}}{d\pi_2^c}\mathbb{E}\left[U'(C_2^c)\frac{\partial C_2^c}{\partial Q^{LC}}\right], \tag{B24}$$

$$\frac{1}{f(X_2)}\frac{dQ^{LC}}{d\pi_2^c} = -M_2^* exp(-\pi_2^c). \tag{B25}$$

This proves Eqns. (31) and (32) for the special case where X_2 is the only shock in the model.

Next, we extend the proof to the case with exchange rate shocks and separate international and domestic endowment shocks. Let f_{jk} denote the probability that domestic real output state X_j and real exchange rate $\mathcal{E}_k$ are realized. Note that we allow domestic output and the real exchange rate to be correlated. We write the probability that output state j is realized as:

$$f_j = \sum_k f_{jk}, \tag{B26}$$

[36]Formally, the proof with a continuous probability density relies on the Calculus of Variations but is otherwise analogous to the discrete probability case.

so f_j continues to be the analogue of the probability density $f(X_2)$ when X_2 follows a continuous distribution. When domestic output takes a discrete set of N values the government's problem simplifies to choosing π_1,π_2, ..., π_N to maximize:

$$\mathbb{E}U = \sum_{j,k} f_{jk} \left(\frac{C_{jk}^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_j^2 \right), \tag{B27}$$

where consumption in state (j, k) is given by

$$C_{jk} = \bar{X}exp(x_j/\bar{X}) - \bar{D}\left(\frac{s}{R^*}\frac{exp(-\pi_j)}{Q^{LC}} + (1-s)\frac{1}{\mathcal{E}_k} \right) \tag{B28}$$

and the LC bond price is given by:

$$\begin{aligned} Q^{LC} &= \mathbb{E}\left[M_2^* exp\left(-\pi_2^c\right)\mathcal{E}_2\right], & \text{(B29)} \\ &= \mathbb{E}\left[\mathbb{E}\left[M_2^*\mathcal{E}_2 \left| X_2 \right.\right] exp\left(-\pi_2^c\right)\right], & \text{(B30)} \\ &= \sum_{j,k} f_{jk}\mathbb{E}\left[M_2^*\mathcal{E}_2 \left| X_2 = X_j \right.\right] exp(-\pi_j), & \text{(B31)} \\ &= \sum_{j} f_j\mathbb{E}\left[M_2^*\mathcal{E}_2 \left| X_2 = X_j \right.\right] exp(-\pi_j). & \text{(B32)} \end{aligned}$$

Here, we have used the law of iterated expectations and the definition of f_j in Eqn. (B26). Taking the derivative of expected domestic consumer utility, $\frac{d\mathbb{E}U}{d\pi_i}$, with respect to the optimal inflation rate in state i gives:

$$\begin{aligned} \frac{d\mathbb{E}U}{d\pi_i} &= \sum_k f_{ik}U'\left(C_{ik}\right)\frac{\partial C_{ik}}{\partial \pi_i} + \frac{\partial}{\partial Q^{LC}}\left(\sum_{j,k} f_{jk}\frac{C_{jk}^{1-\gamma}}{1-\gamma} \right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i & \text{(B33)} \\ &= \sum_k f_{ik}U'\left(C_{ik}\right)\frac{\partial C_{ik}}{\partial \pi_i} + \left(\sum_{j,k} f_{jk}U'\left(C_{jk}\right)\frac{\partial C_{jk}}{\partial Q^{LC}} \right)\frac{dQ^{LC}}{d\pi_i} - f_i\alpha\pi_i. & \text{(B34)} \end{aligned}$$

Setting $\frac{d\mathbb{E}U}{d\pi_i} = 0$ and dividing by the probability f_i gives the inflation first-order condition:

$$\begin{aligned} \alpha\pi_i &= \sum_k \frac{f_{ik}}{f_i}U'\left(C_{ik}\right)\frac{\partial C_{ik}}{\partial \pi_i} + \frac{1}{f_i}\frac{dQ^{LC}}{d\pi_i}\left(\sum_{j,k} f_{jk}U'\left(C_{jk}\right)\frac{\partial C_{jk}}{\partial Q^{LC}} \right), & \text{(B35)} \\ &= \mathbb{E}\left[U'\left(C_{ik}\right)\frac{\partial C_{ik}}{\partial \pi_i}\right| X_2 = X_i \Bigg] + \frac{1}{f_i}\frac{dQ^{LC}}{d\pi_i}\mathbb{E}\left[U'\left(C_{jk}\right)\frac{\partial C_{jk}}{\partial Q^{LC}} \right]. & \text{(B36)} \end{aligned}$$

Taking the derivative of expression (B32) with respect to π_i shows that:

$$\frac{1}{f_i}\frac{dQ^{LC}}{d\pi_i} = -exp(-\pi_i)\mathbb{E}\left[M_2^*\mathcal{E}_2 \left| X_2 = X_i \right.\right]. \tag{B37}$$

If X_2 and $\mathcal{E}_2$ follow continuous distributions we need to replace f_i by $f(X_2)$, and π_i by $\pi_2^c(X_2)$, and C_{ik} by $C_2^c(X_2, \mathcal{E}_2)$ in Eqns. (B36) and (B37). For brevity, we omit the arguments of π_2^c and

C_2^c in the main text. This proves Eqns. (31) and (32) in the main text.[37]

Proof of Inflation First-Order Condition without Commitment

We next prove the no-commitment government's inflation first-order condition Eqn. (30). Without commitment, the government's problem is simply to maximize (B27) subject to (B28) but taking the bond price Q^{LC} as given. If X_2 and $\mathcal{E}_2$ follow discrete probability distributions the first-order-condition becomes:

$$\alpha\pi_i = \sum_k \frac{f_{ik}}{f_i} U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i}. \tag{B38}$$

$$= \mathbb{E}\left[U'(C_{ik}) \frac{\partial C_{ik}}{\partial \pi_i} \middle| X_2 = X_i \right] \tag{B39}$$

If X_2 and $\mathcal{E}_2$ follow continuous probability distributions, we need to replace π_i by $\pi_2^{nc}(X_2)$ and C_{ik} by $C_2^{nc}(X_2, \mathcal{E}_2)$. We again omit the arguments of π_2^{nc} and C_2^{nc}, giving:

$$\alpha\pi_2^{nc} = \mathbb{E}\left[U'(C_2^{nc}) \frac{\partial C_2^{nc}}{\partial \pi_2^{nc}} | X_2 \right]. \tag{B40}$$

Proof of Eqn. (33)

We now prove Eqn. (33) in the main paper. First,

$$\frac{d\mathbb{E}U}{ds} = \frac{d}{ds}\mathbb{E}\left[\frac{C_2^{1-\gamma}}{1-\gamma} - \frac{\alpha}{2}\pi_2^2 \right], \tag{B41}$$

$$= -\alpha\mathbb{E}\left[\pi_2 \frac{d\pi_2}{ds} \right] + \mathbb{E}\left[U'(C_2) \frac{dC_2}{ds} \right]. \tag{B42}$$

Now, recall from Eqns. (14) and (15) that we can write real domestic consumption as:

$$C_2 = X_2 - \frac{\bar{D}}{R^*}\left(sR^{LC} + (1-s)R^{FC} \right). \tag{B43}$$

Because $R^{FC} = R^*$is independent of s it follows that:

$$\frac{dC_2}{ds} = -\frac{\bar{D}}{R^*}\left(R^{LC} - R^{FC} \right) - \frac{s\bar{D}}{R^*}\left(\frac{dR^{LC}}{ds} \right). \tag{B44}$$

Combining Eqns. (B42) and (B44) proves Eqn. (33) in the main paper. Because the government faces a constrained optimization problem of choosing s from the interval $[0, 1]$, a necessary condition for an equilibrium is complementary slackness, that is either $\frac{d\mathbb{E}U}{ds} = 0$ and s is at an interior solution, or $s = 1$ and $\frac{d\mathbb{E}U}{ds} > 0$, or $s = 0$ and $\frac{d\mathbb{E}U}{ds} < 0$.

B.5 Proof of Proposition 1

Government without Commitment

[37] A formal proof with X_2 and $\mathcal{E}_2$ continuous again requires the Calculus of Variations but is otherwise analogous to the discrete case.

We start by log-linearizing the first-order condition for the no-commitment government around $c_2 = 0$ and $\pi_2 = 0$. Recall that the first-order condition for the inflation problem of a government without commitment is given by:

$$\alpha\pi_2 \;=\; \mathbb{E}\left[U'(C_2)\frac{\partial C_2}{\partial \pi_2}\middle| X_2\right]. \tag{B45}$$

Before log-linearizing, we note that the following expressions hold exactly:

$$C_2 \;=\; \bar{X}\exp\left(x_2/\bar{X}\right) - \bar{D}\left((1-s) + s\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right)\right), \tag{B46}$$

$$\frac{\partial C_2}{\partial \pi_2} \;=\; s\bar{D}\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right). \tag{B47}$$

Eqn. (B46) follows from combining Eqns. (14), (15), (16), and (17) and using that in the simplified special case $\mathcal{E}_2$ is constant and equal to one. Eqn. (B47) is the partial derivative of Eqn. (B46) with respect to π_2.

We start the log-linearization by noting that:

$$\begin{aligned} U'(C_2) &= C_2^{-\gamma}, \\ &= \exp(-\gamma c_2) \\ &\approx 1-\gamma c_2. \end{aligned} \tag{B48}$$

We can approximately write period 2 consumption as a log-linear function of domestic output and inflation as follows:

$$\begin{aligned} C_2 &= \bar{X}\exp\left(x_2/\bar{X}\right) - \bar{D}\left((1-s) + s\frac{1}{R^*Q^{LC}}\exp\left(-\pi_2\right)\right) \qquad \text{(B49)} \\ &\approx \bar{X} + x_2 - \bar{D}\left((1-s) + s\left(1-(\pi_2 - \mathbb{E}\pi_2)\right)\right) \\ &= 1 + x_2 + s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right). \end{aligned}$$

Note that we have used the definition that $\bar{X} = \bar{D} + 1$, which ensures that C_2 equals one when all shocks are equal to zero. We have also dropped second- and higher-order terms. It then follows that log consumption approximately equals:

$$c_2 \;\approx\; C_2 - 1 \tag{B50}$$

$$\;\approx\; x_2 + s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right). \tag{B51}$$

Substituting Eqn. (B51) into Eqn. (B48) shows that we can write domestic marginal consumption utility as an approximately log-linear function of domestic output and inflation:

$$U'(C_2) \;\approx\; 1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right). \tag{B52}$$

Also, we have the log-linear approximation

$$\begin{aligned}\frac{\partial C_2}{\partial \pi_2} &= s\bar{D}\frac{1}{R^* Q^{LC}}\exp\left(-\pi_2\right) \\ &\approx s\bar{D}\exp\left(-\left(\pi_2-\mathbb{E}\pi_2\right)\right) \\ &\approx s\bar{D}\left(1-\left(\pi_2-\mathbb{E}\pi_2\right)\right).\end{aligned} \tag{B53}$$

In the special case with no real exchange rate shocks ($\varepsilon_2 = 0$) and only one global output shock ($x_2 = x_2^*$), the conditional expectation on the right-hand side of Eqn. (B45) is trivial and the first-order condition for the no-commitment government has the following log-linear approximation:

$$\begin{aligned}\alpha\pi_2 &= U'\left(C_2\right)\frac{\partial C_2}{\partial \pi_2}, \\ \alpha\pi_2 &\approx s\bar{D}\left(1-\gamma x_2-\gamma s\bar{D}\left(\pi_2-\mathbb{E}\pi_2\right)\right)\left(1-\left(\pi_2-\mathbb{E}\pi_2\right)\right), \\ &\approx s\bar{D}\left(1-\gamma x_2-\gamma s\bar{D}\left(\pi_2-\mathbb{E}\pi_2\right)-\left(\pi_2-\mathbb{E}\pi_2\right)\right),\end{aligned} \tag{B54}$$

where in the last row we have dropped quadratic terms in x_2 and π_2. Solving for π_2 gives the optimal no-commitment inflation policy:

$$\left(\alpha+\gamma\left(s\bar{D}\right)^2+s\bar{D}\right)\pi_2 = s\bar{D}\left(1-\gamma x_2\right)+\left(s\bar{D}+\gamma\left(s\bar{D}\right)^2\right)\mathbb{E}\pi_2. \tag{B55}$$

Because lenders' expectations are rational (B55) implies that $\mathbb{E}\pi_2 = \frac{s\bar{D}}{\alpha}$ and therefore that:

$$\pi_2 = \frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^2+s\bar{D}}\left(1-\gamma x_2\right)+\frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^2+s\bar{D}}\frac{s\bar{D}+\gamma\left(s\bar{D}\right)^2}{\alpha}. \tag{B56}$$

We keep only the lowest-order terms in the debt-to-GDP ratio $\bar{D}$ in the expression (B56). Using the first-order Taylor approximations

$$\frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^2+s\bar{D}} \approx \frac{s\bar{D}}{\alpha}+\mathcal{O}\left(\bar{D}^2\right) \tag{B57}$$

$$\frac{s\bar{D}}{\alpha+\gamma\left(s\bar{D}\right)^2+s\bar{D}}\frac{s\bar{D}+\gamma\left(s\bar{D}\right)^2}{\alpha} \approx 0+\mathcal{O}\left(\bar{D}^2\right) \tag{B58}$$

shows that up to second- and higher-order terms in $\bar{D}$ the inflation policy function (B56) has the following simple form:

$$\pi_2^{nc} \approx \frac{s\bar{D}}{\alpha}-\gamma\frac{s\bar{D}}{\alpha}x_2. \tag{B59}$$

Government with Commitment

We log-linearize the first-order condition for the commitment government around $c_2 = 0$ and $\pi_2 = 0$. Substituting in for $\frac{\partial C_2}{\partial \pi_2}$ from Eqn. (B47) and $\frac{1}{f(X_2)}\frac{dQ^{LC}}{d\pi_2(X_2)} = -\exp(-\pi_2^c(X_2))\mathbb{E}\left[M_2^*\mathcal{E}_2\middle| X_2\right]$ from Eqn. (32), the first-order condition for the inflation problem of a government with commitment is given by:

$$\alpha\pi_2 = \frac{s\bar{D}}{R^* Q^{LC}} \quad \mathbb{E}\left[U'\left(C_2\right)\exp\left(-\pi_2\right)\middle|\, X_2\right] - \mathbb{E}\left[U'\left(C_2\right)\frac{\partial C_2}{\partial Q^{LC}}\right]\exp(-\pi_2)\mathbb{E}\left[M_2^*\mathcal{E}_2\middle|\, X_2\right]. \tag{B60}$$

Note that Eqn. (31) is exact and is the starting point for our log-linearization. In the special case with no real exchange rate shocks ($\varepsilon_2 = 0$) and only one output shock ($x_2 = x_2^*$), Eqn. (19) can be written as:

$$\begin{aligned} \mathbb{E}\left[M_2^*\mathcal{E}_2\middle|\, X_2\right] &= \delta^* \exp(-\gamma^* x_2), \qquad &\text{(B61)}\\ &= \frac{1}{R^*}\exp\left(-\gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2\right). &\text{(B62)} \end{aligned}$$

Taking the partial derivative of Eqn. (B46) with respect to the LC bond price gives the exact expression:

$$\frac{\partial C_2}{\partial Q^{LC}} = \frac{s\bar{D}}{R^*}\frac{\exp(-\pi_2)}{\left(Q^{LC}\right)^2}. \tag{B63}$$

Substituting Eqn. (B62) and Eqn. (B63) into Eqn. (B60), the commitment government's first-order condition can be written as:

$$\begin{aligned} \alpha\pi_2 &= \frac{s\bar{D}}{R^* Q^{LC}}U'\left(C_2\right)\exp\left(-\pi_2\right)\\ &\quad -\frac{s\bar{D}}{\left(R^* Q^{LC}\right)^2}\mathbb{E}\left[U'\left(C_2\right)\exp(-\pi_2)\right]\exp\left(-\pi_2 - \gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2\right). \end{aligned} \tag{B64}$$

The conditional expectations drop out of Eqn. (B64) because we are considering the special case with only one shock. We again use the log-linear expression Eqn. (B52) and log-linearize the last term in Eqn. (B64) to obtain:

$$\begin{aligned} &-\frac{s\bar{D}}{\left(R^* Q^{LC}\right)^2}\mathbb{E}\left[U'\left(C_2\right)\exp\left(-\pi_2\right)\right]\exp\left(-\pi_2 - \gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2\right),\\ \approx\ & -s\bar{D}\mathbb{E}\left[\left(1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right)\right)\exp\left(-\pi_2\right)\right]\exp(2\mathbb{E}\pi_2 - \pi_2 - \gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2),\\ \approx\ & -s\bar{D}\mathbb{E}\left[1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right) - \pi_2\right]\exp(2\mathbb{E}\pi_2 - \pi_2 - \gamma^* x_2 - \frac{1}{2}\left(\gamma^*\right)^2\sigma_x^2),\\ \approx\ & -s\bar{D}\left(1 - \gamma^* x_2 - \left(\pi_2 - \mathbb{E}\pi_2\right)\right). \end{aligned}$$

The remaining terms in the commitment government's first-order condition (B64) are identical to the no-commitment case, so the log-linear approximation to Eqn. (B64) (and hence Eqn. (31)) is given by:

$$\begin{aligned} \alpha\pi_2 &= s\bar{D}\left(1 - \gamma x_2 - \gamma s\bar{D}\left(\pi_2 - \mathbb{E}\pi_2\right) - \left(\pi_2 - \mathbb{E}\pi_2\right)\right)\\ &\quad - s\bar{D}\left(1 - \gamma^* x_2 - \left(\pi_2 - \mathbb{E}\pi_2\right)\right). \end{aligned} \tag{B65}$$

Taking expectations of the left-hand side and right-hand side of Eqn. (B65) and imposing that lenders' expectations are rational shows that $\mathbb{E}\pi_2 = 0$, so optimal log inflation for a government with commitment equals:

$$\pi_2 = \frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2}\left(\gamma^* - \gamma\right)x_2. \tag{B66}$$

Using the first-order Taylor approximation

$$\frac{s\bar{D}}{\alpha + \gamma\left(s\bar{D}\right)^2} = \frac{s\bar{D}}{\alpha} + \mathcal{O}\left(\bar{D}^2\right) \tag{B67}$$

shows that up to second- and higher-order terms in $\bar{D}$ the inflation policy function (B66) has the following simple form:

$$\pi_2 \approx \frac{s\bar{D}}{\alpha}\left(\gamma^* - \gamma\right)x_2 \tag{B68}$$

B.6 Numerical Solution

We solve the model numerically using global projection methods. Our strategy for the numerical solution uses the following strategy:

No-Commitment

1. For any given LC debt share s, we choose the no-commitment inflation function $\pi_2^{nc}(x_2)$ to minimize the error in the government's inflation first-order condition while holding constant the LC debt share using the MATLAB function fminsearch.
2. In an outer loop, we maximize expected domestic consumer utility with respect to the LC debt share, s. For this step, we use the MATLAB function fminbnd over the interval $[0, 1.001]$. The maximization is over optimal expected domestic consumer utility conditional on the LC debt share, s, which we obtain by repeating step 1. above for every value of s.

Commitment

1. For any given LC debt share s, we choose the commitment policy function $\pi_2^c(x_2)$ to maximize expected domestic utility conditional on the LC debt share using the MATLAB function fminsearch.
2. In an outer loop, we maximize expected domestic consumer utility with respect to the LC debt share, s. For this step, we use the MATLAB function fminbnd over the interval $[0, 1.001]$. The maximization is over optimal expected domestic consumer utility conditional on the LC debt share, s, which we obtain by repeating step 1. above for every value of s.

B.6.1 Functional Form

Our numerical procedure considers inflation functions that can be written as a third-order polynomial in x_2:

$$\pi_2^{nc}(x_2) = b_1(s) + b_2(s)x_2 + b_3(s)x_2^2 + b_4(s)x_2^3, \tag{B69}$$

$$\pi_2^{c}(x_2) = c_1(s) + c_2(s)x_2 + c_3(s)x_2^2 + c_4(s)x_2^3. \tag{B70}$$

All coefficients may depend on the LC debt share, s. We use the following vectors as the starting point for our optimization routine:

$$b = [0.0183, -0.5363, 7.9462, -60] \tag{B71}$$

$$c = [0.0028, 0.2061, -5.8417, 20]. \tag{B72}$$

B.6.2 Bond Pricing Function

For any given inflation function, we need to solve for bond prices numerically. To facilitate numerical integration, we first project all exogenous random variables onto x_2 and a shock that is orthogonal to x_2 but is correlated with real exchange rates. We re-write international log real consumption as a component correlated with domestic output plus and an independent shock:

$$x_2^* = \lambda^* x_2 + \eta_2^*, \tag{B73}$$

where we define:

$$\lambda^* = \lambda^{x,x^*} \frac{(\sigma^*)^2}{\sigma_x^2}, \tag{B74}$$

$$\eta_2^* \perp x_2, \tag{B75}$$

$$\left(\sigma_\eta^*\right)^2 = (\sigma^*)^2 - (\lambda^*)^2 \sigma_x^2. \tag{B76}$$

Note that writing the relation between domestic and international endowments as (B73) is consistent with assumptions (23) through (24) in the main paper. That η_2^* is uncorrelated with x_2 is not a new assumption and indeed follows from (23), (24), and the definition λ^*.

For the numerical solution, we use the notation $\rho^* = \lambda^{\varepsilon,x^*}$, so with Eqn. (B73) the log real exchange rate can be written as:

$$\varepsilon_2 = \rho^* x_2^* + e_2 \tag{B77}$$

$$\sigma_e^2 = \sigma_\varepsilon^2 - (\rho^*\lambda^*)^2 \sigma_x^2 - (\rho^*)^2 \left(\sigma_\eta^*\right)^2, \tag{B78}$$

where σ_ε is the standard deviation of the real exchange rate and e_2 is uncorrelated with x_2^* and x_2. We can then write the real exchange rate as a component correlated with log domestic log output plus a shock, e_2^*, that is uncorrelated with domestic output:

$$\varepsilon_2 = (\rho^*\lambda^*)\, x_2 + e_2^*, \tag{B79}$$

$$e_2^* = e_2 + \rho^* \eta_2^*,$$

$$(\sigma_e^*)^2 = \sigma_\varepsilon^2 - (\rho^*\lambda^*)^2 \sigma_x^2.$$

We next use that $1/R^* = \delta^* exp\left(\frac{1}{2}\left(\gamma^*\sigma^*\right)^2\right)$. The ratio of LC bond prices to $1/R^*$ then equals:

$$\begin{aligned}
\frac{Q_1^{LC}}{1/R^*} &= \mathbb{E}_{x_2,e_2^*,\xi_2,\eta_2^*}\left[\exp\left(-\gamma^* x_2^* - \frac{1}{2}\left(\gamma^*\sigma^*\right)^2 - \pi_2 + \rho^* x_2^* + e_2 - \frac{1}{2}\sigma_\varepsilon^2\right)\right] \\
&= \mathbb{E}_{x_2,e_2^*,\xi_2,\eta_2^*}\left[\exp\left(-\gamma^* x_2^* - \frac{1}{2}\left(\gamma^*\sigma^*\right)^2 - \pi_2 + \rho^* x_2^* - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right)\right] && \text{(B80)} \\
&= \mathbb{E}_{x_2,e_2^*,\xi_2}\left[\exp\left(-\left(\theta^* - \rho^*\lambda^*\right)x_2 - \left(\gamma^* - \rho^*\right)\eta_2^* - \pi_2 - \frac{1}{2}\left(\gamma^*\sigma^*\right)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right)\right] && \text{(B81)} \\
&= \mathbb{E}_{x_2,e_2^*,\xi_2}\left[\exp\left(-\left(\theta^* - \rho^*\lambda^*\right)x_2 - \pi_2 + \frac{1}{2}\left(\gamma^* - \rho^*\right)^2\left(\sigma_\eta^*\right)^2 - \frac{1}{2}\left(\gamma^*\sigma^*\right)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right)\right],
\end{aligned}$$

where we define the international lenders' effective risk aversion over domestic output as:

$$\theta^* = \gamma^*\lambda^*. \tag{B82}$$

For any given inflation policy $\pi_2\left(x_2\right)$ it is then relatively convenient to evaluate the following ratio numerically:

$$\frac{Q_1^{LC}}{1/R^* \exp\left(\frac{1}{2}\left(\gamma^* - \rho^*\right)^2\left(\sigma_\eta^*\right)^2 - \frac{1}{2}\left(\gamma^*\sigma^*\right)^2 - \frac{1}{2}\left(\sigma_\varepsilon^2 - \sigma_e^2\right)\right)} = \mathbb{E}_{x_2}\left[\exp\left(-\left(\theta^* - \rho^*\lambda^*\right)x_2 - \pi_2\left(x_2\right)\right)\right]. \tag{B83}$$

We evaluate the expectation (B83) numerically using Gauss-Legendre quadrature with 30 node points, truncating the interval at -6 and +6 standard deviations of x_2.

B.6.3 No-Commitment Policy Function

For a given LC debt share s, we choose the coefficients (b_1, b_2, b_3, b_4) to set the government's inflation first-order condition as close as possible to zero. For any set of coefficients, we evaluate the first-order condition error:

$$Error(x_2) = \mathbb{E}_{e_2^*}\left[\left(-\alpha\pi_2^{nc}(x_2) + (C_2^{nc})^{-\gamma}\frac{dC_2^{nc}}{d\pi_2^{nc}}\right)\middle| x_2\right]. \tag{B84}$$

The expectation (B84) is averaged over e_2^* but conditional on domestic output x_2. At any value of x_2 and e_2^*, no-commitment consumption is evaluated via:

$$\begin{aligned}
C_2^{nc} &= \bar{X}\exp(x_2/\bar{X}) - \bar{D}\left((1-s)\exp\left(-\varepsilon_2 + \frac{1}{2}\sigma_\varepsilon^2\right) + s\frac{1/R^*}{Q_1^{LC}}\exp(-\pi_2^{nc}(x_2))\right). && \text{(B85)} \\
&= \bar{X}\exp(x_2/\bar{X}) - \bar{D}\left((1-s)\exp\left(-\left(\rho^*\lambda^*\right)x_2 - e_2^* + \frac{1}{2}\sigma_\varepsilon^2\right) + s\frac{1/R^*}{Q_1^{LC}}\exp(-\pi_2^{nc}(x_2))\right),
\end{aligned}$$

and the partial derivative of no-commitment consumption with respect to no-commitment inflation is evaluated via:

$$\frac{dC_2^{nc}}{d\pi_2^{nc}} = \bar{D}s\frac{1/R^*}{Q^{LC}}\exp(-\pi_2^{nc}). \tag{B86}$$

Because lenders have rational expectations the LC bond price is evaluated via Eqn. (B83). We evaluate the expectation in Eqn. (B84) over e_2^* numerically using Gauss-Legendre quadrature with 30 nodes and truncation at -6 and +6 standard deviations. We choose the vector of coefficients (b_1, b_2, b_3, b_4) to minimize the expected squared Euler equation error averaged over possible realizations of x_2, that is we minimize $\mathbb{E}_{x_2}\left[Error(x_2)^2\right]$. That is, we minimize the weighted average of the squared Euler equation errors, where each realization of x_2 is weighted by its probability. To take the expectation over x_2, we again use Gauss-Legendre quadrature with 30 nodes and truncation at -6 and +6 standard deviations.

In the outer loop, we then maximize expected utility $\mathbb{E}_{x_2,e_2^*}\left[\left(-\frac{\alpha}{2}\left(\pi_2^{nc}\right)^2+\frac{C_2^{nc,1-\gamma}}{1-\gamma}\right)\right]$ in Eqn. (B84) over s, where for any s the coefficients (b_1, b_2, b_3, b_4) are found as described above.

B.6.4 Commitment Policy Function

For a given LC debt share s, we choose the commitment inflation policy function coefficients (c_1, c_2, c_3, c_4) to maximize the expectation:

$$\mathbb{E}_{x_2,e_2^*}\left[\left(-\frac{\alpha}{2}\left(\pi_2^{c}\right)^2+\frac{C_2^{c,1-\gamma}}{1-\gamma}\right)\right], \tag{B87}$$

where we evaluate commitment consumption numerically:

$$C_2^c \quad = \quad \bar{X}\exp(x_2/\bar{X}) - \bar{D}\left((1-s)\exp\left(-\left(\rho^*\lambda^*\right)x_2 - e_2^* + \frac{1}{2}\sigma_\varepsilon^2\right) + s\frac{1/R^*}{Q_1^{LC}}\exp(-\pi_2^c(x_2))\right). \tag{B88}$$

and LC bond prices update with the commitment inflation policy function through (B83). All expectations are again evaluated numerically using Gauss-Legendre quadrature using the same grid points as before.

In the outer loop, we maximize $\mathbb{E}_{x_2,e_2^*}\left[\left(-\frac{\alpha}{2}\left(\pi_2^{c}\right)^2+\frac{C_2^{c,1-\gamma}}{1-\gamma}\right)\right]$ over s, where for any s the coefficients (c_1, c_2, c_3, c_4) are found as described above.

B.6.5 Model Moments

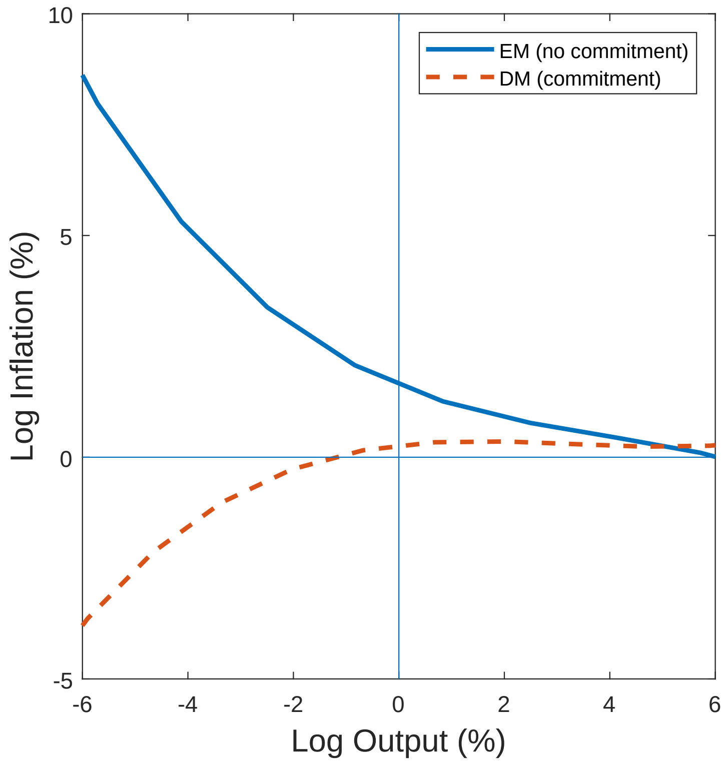
We use Gauss-Legendre quadrature to evaluate inflation moments numerically. For both x_2 and e_2^*, we use 30 nodes and truncate the interval at -6 and +6 standard deviations. We evaluate average inflation, the bond-stock beta and the LC bond risk premium numerically as:

$$\begin{aligned}
\mathbb{E}^{model}\pi_2 &= \mathbb{E}_{x_2,e_2^*}\pi_2, && \text{(B89)}\\
\beta^{model}(bond_i, stock_i) &= \frac{-1}{\lambda^{m,x}}\frac{\mathbb{E}_{x_2,e_2^*}\left[\left(\pi_2-\mathbb{E}\pi_2\right)x_2\right]}{\sigma_x^2}, && \text{(B90)}\\
RP^{LC,model} &= \log\mathbb{E}_{x_2,e_2^*}\left[\exp(-\pi_2)\right] - \log Q^{LC} - r^*. && \text{(B91)}
\end{aligned}$$

B.6.6 Plotting the Inflation Policy Functions

Figure B1 shows inflation as a function of period 2 log domestic output. Consistent with the intuition from Proposition 1, DM inflation decreases in the worst states of the world, thereby providing international lenders with safe assets. A government with commitment optimally adopts

Figure B1: Inflation Policy Functions



Note: This figure shows log inflation π_2 against log output x_2 both in annualized %, in the calibrated model. The solid blue lines indicate the EM calibration, while the dashed red lines indicate the DM calibration.

pro-cyclical inflation, selling insurance to international lenders and earning the risk premium. This is similar to the problem studied in Farhi and Maggiori (2018) with a risk-neutral government and risk-averse lenders. By contrast, EM inflation increases in the worst states of the world. Intuitively, EM governments cannot commit to limiting their own consumption smoothing and instead have an incentive to use inflation in the worst states of the world to smooth domestic consumption fluctuations.

B.7 Calibration Robustness

B.7.1 Risk-Neutral Investors

The following table BI shows the calibration moments for an alternative calibration, that sets risk aversion parameters to conventional values in the real business cycle literature ($\gamma^* = 0, \gamma = 2$). All other parameters are as listed in Table V. As described in Section B, under this alternative calibration the bond-stock beta is somewhat higher for DM (commitment) countries than for EM (no commitment) countries, so it cannot match our main empirical evidence that bond-stock betas are negatively correlated with LC debt shares across countries. Of course, LC bond risk premia are also zero under this alternative calibration, because international lenders are risk-neutral.

Table BI: Empirical and Model Moments - Risk Neutral Calibration

	EM (no commitment)		DM (commitment)		EM-DM	
	Data	Model	Data	Model	Data	Model
Average Inflation	3.92%	1.46%	1.73%	0.00%	2.20%	1.46%
LC Bond-Stock Beta	0.07	0.01	-0.10	0.02	0.17	-0.02
LC Debt Share	0.55	0.43	0.90	1.00	-0.35	-0.57
LC Bond RP	3.15%	0.00%	1.53%	0.00%	1.62%	0.00%

Note: All moments are in annualized natural units. This table is analogous to Table VI in the main paper, except that $\gamma^* = 0$ and $\gamma = 2$. All other model parameters for the EM and DM calibrations are given in Table V. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (B10). The model LC bond risk premium in % is computed according to Eqn. (39).

B.7.2 Separate EM and DM Local-International Endowment Loadings

We now verify that calibration results are unchanged if we match the domestic-international endowment loadings to the data separately for emerging and developed markets. We set $\lambda^{x,x^*} = 0.87$ for EMs and $\lambda^{x,x^*} = 0.97$ for DMs to match the average slope coefficients of domestic output growth with respect to US consumption growth averaged separately for EM and DM data. All other parameter values are as listed in Table V. Table BII shows that the model moments are qualitatively and quantitatively unchanged compared to Table VI in the main paper.

Table BII: Model Moments with Separate Local-International Endowment Loadings

	EM (no commitment)		DM (commitment)		EM-DM	
	Data	Model	Data	Model	Data	Model
Average Inflation	3.92%	2.12%	1.73%	0.00%	2.20%	2.11%
LC Bond-Stock Beta	0.07	0.15	-0.10	-0.06	0.17	0.21
LC Debt Share	0.55	0.38	0.90	0.96	-0.35	-0.58
LC Bond RP	3.15%	4.27%	1.53%	2.03%	1.62%	2.24%

Note: All moments are in annualized natural units. Model parameters for the EM and DM calibrations are given in Table V, except for the local-global endowment loadings, which we set to set to $\lambda^{x,x^*} = 0.87$ for EMs and $\lambda^{x,x^*} = 0.97$ for DMs. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (B10).

B.7.3 Separate EM and DM Exchange Rate Processes

We now verify that the calibration results are qualitatively and quantitatively unchanged if we calibrate EM and DM real exchange rate processes separately to the data. To match the data moments averaged separately over EMs and DMs, we set $\sigma_\varepsilon = 10.4\%$ and $\lambda^{\varepsilon,x^*} = 1.33$ for the EM calibration and $\sigma_\varepsilon = 11.4\%$ and $\lambda^{\varepsilon,x^*} = 1.56$ for the DM calibration. All other parameter values are as listed in Table V. The resulting model moments are shown in Table BIII.

Table BIII: Model Moments with Separate Exchange Rate Processes

	EM (no commitment)		DM (commitment)		EM-DM	
	Data	Model	Data	Model	Data	Model
Average Inflation	3.92%	2.07%	1.73%	0.00%	2.20%	2.06%
LC Bond-Stock Beta	0.07	0.15	-0.10	-0.05	0.17	0.20
LC Debt Share	0.55	0.37	0.90	0.91	-0.35	-0.54
LC Bond RP	3.15%	3.95%	1.53%	2.53%	1.62%	1.42%

Note: All moments are in annualized natural units. Model parameters for the EM and DM calibrations are given in Table V, except for the exchange rate processes, which we calibrate separately to the data in this table. We set $\sigma_\varepsilon = 10.4\%$ and $\lambda^{\varepsilon,x^*} = 1.33$ for the EM calibration and $\sigma_\varepsilon = 11.4\%$ and $\lambda^{\varepsilon,x^*} = 1.56$ for the DM calibration. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (B10).

B.7.4 Varying the DM Inflation Cost Parameter

We now verify the robustness of our calibration results to choosing different inflation cost parameters for the DM calibration. In our baseline calibration, the inflation cost parameter, α, is pinned down by the average difference in inflation between EMs and DMs in the data. In our baseline calibration, we choose the same inflation cost parameter for EMs and DMs for symmetry and to focus on the effect of credibility, which also varies across EM and DM calibrations. However, it appears plausible that the inflation cost of DMs is different from EMs. The DM inflation cost could be higher if DM policy makers assign a higher cost to inflation. Or it could be lower, if DM institutions are better able to smooth out frictions caused by inflation.

Here, we verify that the calibration results are similar for a range of values for the DM inflation cost parameter, α^{DM}. We consider a wide range of values for α^{DM}, setting it to one half and twice the baseline value of $\alpha = 4.28$. All other parameter values are set to the DM values in Table V. The resulting model moments in Table BIV show that DM model moments are largely insensitive to α^{DM}. Average inflation is equal to zero – the optimal level in the model – for all values of α, because a government with full commitment always chooses average inflation equal to the optimal level. The LC debt share is close to 0.90 for a wide range of inflation cost parameters, and the bond-stock beta varies within a relatively narrow range from -0.03 to -0.09.

Table BIV: Model Robustness to Different Inflation Costs

	DM Data	Baseline $\alpha = 4.28$	Low Inflation Cost $\alpha = 2.14$	High Inflation Cost $\alpha = 8.56$
Average Inflation	1.73%	0.00%	0.00%	0.00%
LC Bond-Stock Beta	-0.10	-0.05	-0.09	-0.03
LC Debt Share	0.90	0.91	0.92	0.90
LC Bond RP	1.53%	2.22%	1.90%	2.46%

Note: All moments are in annualized natural units. Model parameters are given by the DM calibration in Table V, except for the inflation cost, α, which is listed in the column header. Model average inflation is the unconditional average of level inflation. The model bond-stock beta is computed according to Eqn. (B10).